

# How Misconceptions Affect Formal Physics Problem Solving: Model-Based Predictions and Empirical Observations

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## Abstract

One important finding in physics education is that very often students enter physics courses with misconceptions about the domain. An often raised, but hardly ever thoroughly investigated question is whether and how students' misconceptions in physics come into play in solving formal textbook problems which ask for a precise quantitative solution. We developed a cognitive computer model of the role qualitative physics knowledge plays in formal physics problem solving. On the basis of the model it cannot only be hypothesized where misconceptions might come into play during formal physics problem solving, but also which correct qualitative physics knowledge should be applied instead in order to guide the use of quantitative physics knowledge efficiently and successfully. In particular, the model predicts that the application of misconceptions prevents the results of qualitative problem analyses from being exploited to construct additionally required formal, quantitative physics knowledge. An empirical investigation confirmed that misconceptions frequently affect formal physics problem solving in the way predicted by the model. Commonly, subjects who applied misconceptions during problem solving reached an impasse when they tried to express the results of their qualitative problem analyses in quantitative terms. Most of the subjects were not able to resolve such an impasse successfully.

## Introduction

One of the major findings in research on science education in formal sciences such as physics is that students before instruction develop preconceptions about the phenomena that physicists explain (for a bibliography see Pfundt & Duit, 1991). Because very often these preconceptions differ from the concepts taught, they are frequently named misconceptions.

Though misconceptions have received much attention in research on science education, the major work consists in documenting as many of them as possible. Commonly, to pinpoint misconceptions, problems are posed to students which can be solved by exclusively making use of conceptual, qualitative physics knowledge. Figure 1a shows a problem that is exemplary for the problems which have frequently been taken advantage of by investigators of misconceptions in physics (e.g., Clement, 1982; McCloskey, 1983). To solve such a problem it is not required to apply formal, quantitative physics knowledge which relies on

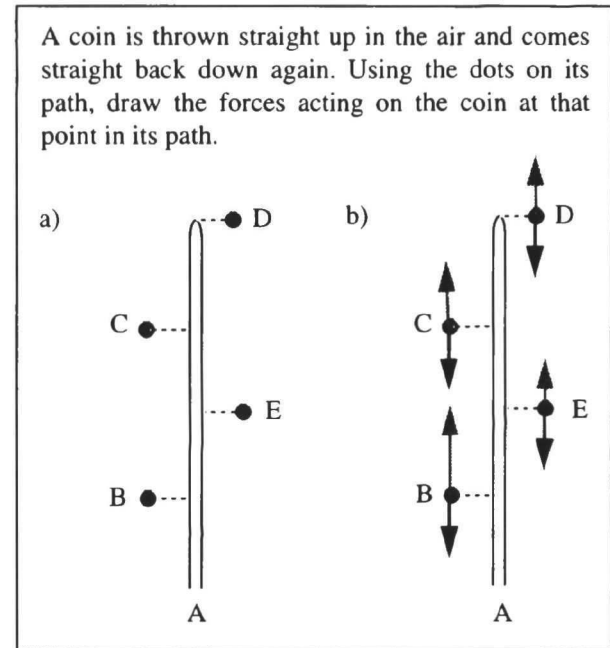


Figure 1: A physics problem to pinpoint misconceptions (a) and a frequently observed incorrect solution (b).

mathematical formalisms such as algebraic equations.

Figure 1b displays a frequently observed incorrect solution to the problem shown in Figure 1a. In addition to the gravitational force, it erroneously involves an upward-pointing, decreasing, and non-existing force to account for the object's motion. Since this misconception resembles in many ways the concept of impetus as it was discussed during the middle ages by Philoponus and others (for a discussion see Franklin, 1976; Szabo, 1976), it is usually named impetus concept, one of the most prominent misconceptions in classical mechanics. Typically, the impetus concept is applied to moving objects characterized by the absence of a proper force in the direction of motion.

In research on physics education, much less attention has been given to describing whether and how misconceptions such as the impetus concept also come into play during formal, quantitative physics problem solving and to modeling the problem solving of successful students. This

neglect might be rooted in an assumption that seems to be considerably widespread in the physics education community.

It is often assumed that conceptual, qualitative physics knowledge is a subset of formal, quantitative physics knowledge. Thus, mastering quantitative physics should cause mastery of qualitative physics and hence remedy qualitative physics errors including those based on misconceptions. As a consequence, instruction frequently focuses on quantitative physics knowledge in order to set up conditions on which misconceptions are not applicable any more. Though we have not found explicit statements expressing this assumption, we have inferred it from how the relationship between qualitative and quantitative physics knowledge is frequently addressed in the literature as well as in physics textbooks.

In recent studies, however, we were able to show that qualitative physics knowledge is on no account a mere subset of quantitative physics knowledge (Ploetzner, 1993; Ploetzner & VanLehn, in preparation). Instead, both kinds of knowledge encode complementary information. In particular, successful problem solving approaches frequently demand the coordinated use of both kinds of knowledge. Due to these findings it is not safe to assume that misconceptions do not affect formal physics problem solving.

We developed a cognitive computer model of the role qualitative physics knowledge plays in formal, quantitative problem solving. On the basis of this model it cannot only be hypothesized where misconceptions might come into play, but also which correct qualitative physics knowledge should be applied instead.

After a brief overview of the cognitive computer model has been presented, the model is taken advantage of to predict how misconceptions might affect formal physics problem solving. Thereafter an empirical study is described bringing to the fore how students' formal problem solving is influenced by misconceptions. The text concludes with a short discussion of the findings.

## **The Coordinated Use of Qualitative and Quantitative Physics Knowledge**

To better understand how qualitative and quantitative physics knowledge complements each other in formal, quantitative problem solving, we developed a cognitive computer model which simulates how the use of correct qualitative and quantitative knowledge can be coordinated to solve textbook problems in physics successfully and efficiently (Ploetzner & Spada, 1993). The problems are taken from the domain of one-dimensional motion with constant acceleration. The knowledge investigated is made up of qualitative and quantitative physics knowledge about various concepts in dynamics (e.g., gravitational, normal, and friction force) and kinematics (e.g., position, velocity, and acceleration).

The model has been implemented in Prolog by taking advantage of an equation-based representation language sim-

ilar to the representation language employed by VanLehn, Jones and Chi (1992). Qualitative physics knowledge encodes information such as the abstractions to be considered in a certain problem situation, the conditions on which physics concepts can legitimately be applied, the attributes possessed by physics concepts and the values concept attributes might have. Quantitative physics knowledge encodes precise functional relationships between the physics concepts by means of algebraic and vector-algebraic equations.

The main focus of the model is on how the results of qualitative reasoning can be taken advantage of during subsequent quantitative problem solving. The model simulates characteristic differences in the problem solving performance of those subjects who coordinate their qualitative and quantitative physics knowledge and those subjects who do not. Especially, on the basis of the model it can be shown that efficient and successful problem solving in the application domain requires the coordinated use of qualitative and quantitative physics knowledge.

Coordination can take place within the model in two different ways. Firstly, by drawing on vector-algebraic knowledge, the results of qualitative problem analyses are expressed in algebraic terms to construct additional quantitative physics knowledge not available to the model beforehand. It may algebraically specify the resultant force on an object whose motion has to be analyzed, for example. In many cases, such additionally constructed quantitative knowledge is a necessary prerequisite for being able to solve the posed problems.

Secondly, the outcome of qualitative reasoning is exploited to constrain the use of already available quantitative knowledge. This kind of coordination makes quantitative problem solving more efficient because how quantitative physics knowledge is used depends more on the way how the problem is represented qualitatively and less on what is the unknown quantity. For further details about the cognitive computer model the interested reader is referred to Ploetzner and Spada (1993).

The role qualitative domain knowledge plays in problem solving has been previously investigated with respect to other domains as well. For example, Ohlsson and Rees (1991) developed a cognitive computer model which simulates how knowledge about qualitative domain principles in mathematics can be taken advantage of to constrain problem solving.

## **Predictions**

To predict how misconceptions such as the impetus concept affect formal physics problem solving the qualitative knowledge available to the model has been extended with incorrect qualitative knowledge which encodes the misconceptions under scrutiny. Subsequently, the model has been applied to a number of formal physics problems and

its change in problem solving performance has been analyzed.

In the model, the application of misconceptions during formal physics problem solving affects both kinds of coordination as described above. With respect to the first kind of coordination, the application of misconceptions such as an impetus concept results in qualitative problem representations which cannot be utilized to construct additionally required quantitative knowledge. This is mainly due to the fact that misconceptions have no formal, quantitative counter parts. If qualitative problem representations which involve misconceptions have to be expressed algebraically, an impasse is reached. In the model, such an impasse can only be overcome by dropping the misconceptions and by applying correct qualitative knowledge instead. Otherwise, additionally required quantitative physics knowledge cannot be constructed and problem solving fails.

With respect to the second kind of coordination, the application of misconceptions results in qualitative problem representations which can only partially be exploited to constrain the subsequent use of quantitative physics knowledge. In solving simple problems which do not require the first kind of coordination this (merely) leads to less efficient problem solving behavior.

Thus, on the basis of the model it is predicted that the application of misconceptions by humans during formal physics problem solving affects their ability to coordinate their qualitative and quantitative physics knowledge in the ways just described.

Since the problems considered in this text do require the first kind of coordination, the question of efficiency is not considered any further.

## Empirical Investigations

In order to examine how students coordinate their qualitative and quantitative physics knowledge in formal problem solving, we conducted an empirical study which comprised 28 subjects (6 females and 22 males) who were students of a technically oriented high school. In this type of school, adults may receive a degree which qualifies for university entrance after they have already completed a first professional training. The grade corresponded to grade 13. The subjects' age ranged between 20 and 28 years. At the time the investigation took place, the physics of particle motion in one dimension had been taught to the students about 9 months before.

In the course of the study, the subjects had to work on five textbook problems. All problems asked for a precise quantitative solution. During problem solving, the subjects had available two tables. The first table showed all the quantitative physics relevant to the posed problems. The second table presented a short summary of how to resolve vectors for their components. The subjects were urged to use only the provided materials, to apply no energy principles, to make notes of all their attempts to solve a problem as

detailed as possible and to work on the problems as in a classroom examination.

In the following, only those problems posed to the subjects will be considered which are characterized by the absence of a force in the direction of motion (cf. Table 1). Since the impetus concept could be applied to each of these problems, they are of special interest with respect to the question of how students' misconceptions affect their performance in formal problem solving.

Table 1: Three problems characterized by the absence of a force in the direction of motion.

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**Problem 1:** What is the minimum stopping distance for a car travelling along a flat horizontal road with the velocity  $v = 30 \text{ m/s}$ , if the coefficient of friction between tires and road is  $f = 0.6$ ?

**Problem 2:** A coin is tossed straight up into the air with the velocity  $v = 7 \text{ m/s}$ . How far up does the coin go until its velocity is reduced to  $v = 3 \text{ m/s}$ ?

**Problem 3:** A block of mass  $m = 10 \text{ kg}$  is projected up an inclined plane with the velocity  $v = 5 \text{ m/s}$ . The plane is inclined at an angle  $\alpha = 30^\circ$ . How far up the plane does the block go, if the coefficient of friction between block and plane is  $f = 0.3$ ?

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Table 2 shows the observed number of correct and incorrect solutions to each problem. In many cases subjects were not able to come up with a final solution but stopped working on the problem. The application of an impetus concept has been diagnosed in 7 cases with respect to Problem 1, in 5 cases with respect to Problem 2 and in 16 cases with respect to Problem 3. Generally, the application of an impetus concept has been diagnosed, whenever a subject drew or verbally referred to a force in the direction of motion. One subject applied an impetus concept to all three problems, five subjects applied an impetus concept to Problem 1 and 3, four subjects to Problem 2 and 3, one subject only to Problem 1 and six subjects only to Problem 3.

Table 2: Observed number of correct and incorrect solutions.

	Problem			$\Sigma$
	1	2	3	
Correct solutions	14	7	9	30
Incorrect solutions	4	13	14	31
No solution	10	8	5	23

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Further misconceptions which have been diagnosed (cf. also Halloun & Hestenes, 1985) are the beliefs that an

object subjected to a constant force moves with constant velocity (3 applications) and that acceleration and velocity are the same concepts (4 applications). Unlike what is commonly believed, these results clearly demonstrate that students frequently make use of their misconceptions such as an impetus concept even in a formal problem solving setting.

However, had the application of an impetus concept any consequences with respect to the correctness of the problem solutions? Table 3 reveals that the number of correct problem solutions decreased significantly, if an impetus concept had been applied ( $\chi^2 = 13.26$ ;  $df = 2$ ;  $p < .01$ ).<sup>1</sup> The use of misconceptions obviously affected formal problem solving in that it frequently led to incorrect or no problem solutions at all.

Table 3: How the use of an impetus concept affects the correctness of problem solutions.

	correct solution	incorrect solution	no solution	$\Sigma$
impetus applied	3	12	13	28
no impetus applied	27	19	10	56
$\Sigma$	30	31	23	84

As predicted by the model, in most of the cases, subjects who applied an impetus concept encountered severe difficulties in their problem solving attempts exactly when the results of their (incorrect) qualitative problem analyses had to be coordinated with the use of their quantitative physics knowledge. In 25 cases (89%) in which an impetus concept has been applied, subjects were not able to express the results of their (incorrect) qualitative reasoning algebraically and reached an impasse. 12 subjects resolved this impasse by means of illegal algebraic transformations, for example, leading to incorrect problem solutions.

13 subjects, however, stopped making any further effort to solve the respective problem. Figure 2 exemplifies such an observed problem solving behavior with respect to Problem 3. Apparently, the subject holds an impetus concept. It constructs a free-body diagram which, among other inadequacies, visualizes a non-existing force in the direction of motion. The subject refers to this force by the symbol  $F_{push}$ . In accord with the free-body diagram, an incorrect resultant force is subsequently derived. Thereafter, the subject writes down various kinematics laws, but finally fails to derive the problem's solution because she is not able to determine the blocks acceleration.

1. This is not to say that the problems would have been solved correctly otherwise. Many subjects made multiple errors during problem solving such as applying an impetus concept and neglecting some relevant proper physics concept.

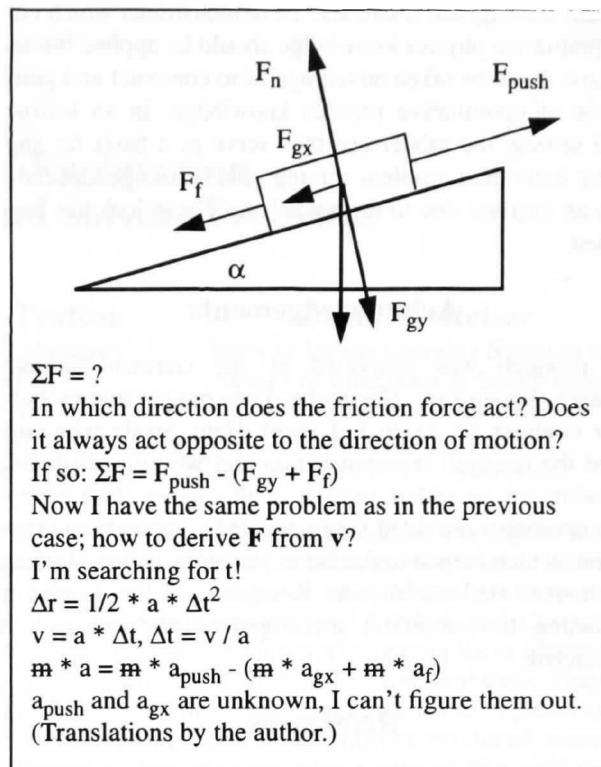


Figure 2: How a subject failed by applying an impetus concept.

Only in 3 cases subjects who applied an impetus concept came up with correct problem solutions. After one subject failed to algebraically express an arrow drawn in the direction of motion, he simply dropped the impetus concept represented by that arrow and successfully resumed its problem solving attempts. Two subjects determined the correct problem solutions by treating the assumed impetus concepts algebraically as if they were resultant forces.

## Conclusions

On the basis of a cognitive computer model it has been predicted that misconceptions also might come into play when solving physics problems which ask for precise quantitative solutions. In particular, it has been predicted that the use of misconceptions makes it difficult to exploit qualitative problem representations in constructing additionally required quantitative physics knowledge. An empirical investigation confirmed the model-based prediction. Students who applied misconceptions reached an impasse in their problem solving attempts when they tried to express the results of their qualitative reasoning in quantitative terms. In most of the cases the students were not able to resolve such an impasse successfully.

By means of the model, however, it cannot only be hypothesized how misconceptions affect formal physics

problem solving, but it can also be demonstrated which correct qualitative physics knowledge should be applied instead and how it can be taken advantage of to construct and guide the use of quantitative physics knowledge. In an instructional setting, the model can thus serve as a basis for supporting individual problem solving (and learning) especially when an impasse due to the use of misconceptions has been reached.

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