Continuity Effect and Figural Bias in Spatial Relational Inference

Markus Knauff (knauff@cognition.iig.uni-freiburg.de)
University of Freiburg, Center for Cognitive Science, Friedrichstr. 50, D – 79098 Freiburg i. Br., Germany
Reinhold Rauh (reinhold@cognition.iig.uni-freiburg.de)
University of Freiburg, Center for Cognitive Science, Friedrichstr. 50, D – 79098 Freiburg i. Br., Germany
Christoph Schlieder (schliede@informatik.tu-muenchen.de)
Technical University of Munich, Department of Computer Science, Arcisstr. 21, D – 80290 München, Germany
Gerhard Strube (strube@cognition.iig.uni-freiburg.de)
University of Freiburg, Center for Cognitive Science, Friedrichstr. 50, D – 79098 Freiburg i. Br., Germany

Abstract

Two experiments on spatial relational inference investigated effects known from relational and syllogistic reasoning. (1) Continuity effect: n-term-series problems with continuous ($W\ r_1\ X, X\ r_2\ Y, Y\ r_3\ Z$) and semi-continuous ($X\ r_2\ Y, Y\ r_3\ Z, W\ r_1\ X$) premise order are easier than tasks with discontinuous order ($Y\ r_3\ Z, W\ r_1\ X, X\ r_2\ Y$). (2) Figural bias: the order of terms in the premises ($X\ r\ Y, Y\ r\ Z$ or $Y\ r\ X, Z\ r\ Y$) effects the order of terms in the conclusion ($X\ r\ Z$ or $Z\ r\ X$). In the first experiment subjects made more errors and took more time to process the premises when in discontinuous order. In the second experiment subjects showed the general preference for the term order $Z\ r\ X$ in the generated conclusions, modulated by a "figural bias": subjects used $X\ r\ Z$ more often if the premise term order was $X\ r\ Y, Y\ r\ Z$, whereas $Z\ r\ X$ was used most often for the premise term order $Y\ r\ X, Z\ r\ Y$. Results are discussed in the framework of mental model theory with special reference to computational models of spatial relational inference.

Introduction and Related Work

The general scheme of an important class of tasks studied in the psychology of reasoning are the so called *n-term-series problems*, in which subjects have to find a conclusion on the basis of given premises. In the special case of a spatial *three-term series problem (3ts-problem)*, two spatial relational terms $X\ r_1\ Y$ and $Y\ r_2\ Z$ are given as premises (Johnson-Laird, 1972). The goal is to find a conclusion $X\ r_3\ Z$ that is consistent with the premises. In a *four-term series problem (4ts-problem)*, three premises $W\ r_1\ X$, $X\ r_2\ Y$ and $Y\ r_3\ Z$ are given, and three relations not explicitly given, namely $W\ r_4 Y$, $X\ r_5\ Z$ and $W\ r_6\ Z$, can be inferred (Rauh & Schlieder, 1997).

Recent theoretical accounts of human reasoning have been proposed by Rips (1994), who views reasoning as a rule-based proof procedure, and by mental model theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991).

Our previous work shows that particular spatial relational inference lends itself well to an explanation based on the construction of spatial mental models (Knauff, Rauh, & Schlieder, 1995; Rauh, Schlieder, 1997; overview in: Knauff, Rauh, Schlieder & Strube, 1998).

The common denominator of mental model accounts for explaining the cognitive processes underlying such inferences is the conception of reasoning as a process in which, at first, unified mental representations of the given premises are generated and then, due to the fact that this information can be ambiguous, alternative models of the premises are sequentially generated and inspected.

In the *construction phase* reasoners use their knowledge about the semantics of spatial expressions to construct an internal model of the situation that the premises describe. This is the stage of the reasoning process in which the premises are integrated into a unified mental model.

In the *inspection phase*, a parsimonious description of the mental model is constructed, including a preliminary conclusion. In other words, the mental model is inspected to find relations which are not explicitly given.

In the *variation phase*, people try to find alternative models of the premises in which the conclusion is false. If they cannot find such a model, the conclusion must be true.

Evidence that people ***construct*** integrated representations of the given premises in the sense of a mental model has been found through the investigation of premise order. The reported result is often called "continuity effect" or "order of premises effect" in the literature (Evans, Newstead, & Byrne, 1993).

Important hints with respect to the ***inspection*** of mental models, can be collected in the investigation of term order in the premises. The reported result in the area of syllogistic reasoning is often called "figural effect" or "figural bias" (Johnson-Laird & Bara, 1984).

Continuity Effect

Ehrlich and Johnson-Laird (1982), for instance, gave subjects relational 4ts-problems and the three premises $W\ r_1\ X$, $X\ r_2\ Y, Y\ r_3\ Z$ were presented in continuous ($W\ r_1\ X, X\ r_2\ Y, Y\ r_3\ Z$), semi-continuous ($X\ r_2\ Y, Y\ r_3\ Z, W\ r_1\ X$), and discontinuous order ($Y\ r_3\ Z, W\ r_1\ X, X\ r_2\ Y$). Subjects had to infer only the conclusion $X\ r_4\ Z$. The dependent measures were the error rates and premise processing times for each kind of premise order.

The results of Ehrlich and Johnson-Laird (1982; Exp. 3) support the prediction of mental model theory that continuous order (37% errors) should be easier than discontinuous order (60% errors), and there is no significant difference between continuous and semi-continuous (39% errors) tasks.

Mental model theory explains these results as an effect of the difficulty of integrating the information from the premises. In the continuous and semi-continuous orders, it is possible to integrate the information of the first two premises directly at the beginning into one model, whereas when they are presented with the discontinuous order subjects must wait for the third premise in order to integrate the information in the premises into a unified representation. Before they get this information they have to temporarily store the information from the first and second premise separately, making the task much harder.

Experiment 1 below was conducted to investigate the effect of premise order in spatial relational inference.

Figural Bias

When investigating the effect of premise order an obvious question is whether there is a similar effect for the order of objects (terms) inside the premises. This has been done extensively in the area of syllogistic reasoning and researchers have come up with an extremely reliable and very robust effect that is called the "figural effect" or "figural bias" (Hunter, 1957; De Soto, London & Handel, 1965; Trabasso, Riley, & Wilson, 1975). We explain this effect according to an experiment on relational inference by Johnson-Laird and Bara (1984). They gave subject 3ts-problems of the following types:

Type 1:	Type 2:
X is related to Y	Y is related to X
Y is related to Z	Z is related to Y

The result was, that in problems of Type 1 subjects tend to spontaneously generate more conclusions in the form "*X is related to Z*" than the other correct conclusion "*Z is related to X*", whereas they tend to generate more conclusions in the form "*Z is related to X*" for problems of Type 2. According to the rule-based, mental proof theory, the surface features of the premises determine the figural effect (Rips, 1994). However, Johnson-Laird and Bara (1984) explained the "figural effect" according to mental model theory. They assumed that the integration of the premises in working memory is more difficult in Type 2 problems because of the need to bring the Y term into the middle. According to this view, the construction of a mental model is easier for premises that have the repeated term as first term in the next premise. In this case, the information of the given premises can be integrated immediately and no cognitive resources are needed for mental operations that bring the middle term into the middle.

Experiment 2 below was conducted to find out whether a figural effect can be found for the spatial domain as well.

Experiment I: Order of Premises Effect

The computer-aided experiment was separated into three blocks: a *definition*, a *learning*, and an *inference phase*. The reasons for the procedure are discussed extensively in Knauff, Rauh, and Schlieder (1995). The main idea was to distinguish between conceptual and inferential aspects of the used spatial relations and to refer the obtained results to the pure inference process holding constant the conceptual aspects.

Subjects

36 paid students (18 female, 18 male) of the University of Freiburg, ranging in age from 21 to 33 years.

Material

As in our previous experiments (for example: Knauff, Rauh, & Schlieder, 1995; Rauh & Schlieder, 1997) we used a system of relations that was introduced by Allen (1983). Although Allen's theory was originally developed in the area of temporal reasoning it has triggered numerous research enterprises in spatial reasoning as well (Freksa, 1991). The main reason for preferring this set of relations over natural language expressions like *left–right, before–behind, east–west, north–south* (e.g. Byrne & Johnson-Laird, 1989; Ehrlich & Johnson-Laird, 1982; Johnson-Laird & Byrne, 1991; Maki, 1981; Mani & Johnson-Laird, 1982) is the possibility to formulate a model-theoretic semantics of the relations, which allows the exact determination of what counts as a model and what not (e.g., Nebel & Bürckert, 1995).

Allen's calculus is based on intervals, qualitative relations between these intervals, and an algebra for reasoning about relations between these intervals. Firstly, Allen (19983) denotes thirteen qualitative jointly exhaustive and pairwise disjoint relations describing the relative position of two intervals on a line: *before (<)* and its converse *after (>)*, *meets (m)* and *met by (mi)*, *overlaps (o)* and *overlapped by (oi)*, *finishes (f)* and *finished by (fi)*, *during (d)* and *contains (di)*, *starts (s)* and *started by (si)*, and *equal (=)* that has no converse. Figure 1 gives pictorial examples of these relations[1].

Secondly, Allen introduced a reasoning algorithm based on these relations. For instance, if the system receives the information that *X meets Y* and *Y is during Z* it is computed that the following relations between X and Z are possible: *X overlaps Z* or *X is during* Z or *X starts with Z*. The set of all possible conclusions that has $X\ r_1\ Y$ and $Y\ r_2\ Z$ as its premises can be denoted as $c\ (r_1\ r_2)$. Since Allen's theory contains thirteen relations, we get 144 compositions $c\ (r_1\ r_2)$, omitting the trivial "*equal*" relation. This compositions can be used directly as 3ts-problems and it is also possible to construct 4ts-problems on the basis of the 13 relations.

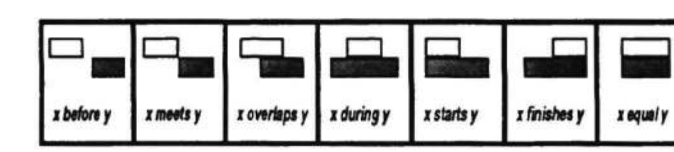

Figure 1: The possible qualitative relations that can hold between two intervals (Allen, 1983).

Method and Procedure

In the *definition phase*, subjects read descriptions of the locations of a red and a blue interval using the 13 qualitative relations (in German). Each verbal description was presented with a short commentary about the location of the beginnings and endings of the two intervals and a picture with a red and blue interval that matched the description.

The *learning phase* consisted of blocks of trials, where subjects were presented with the one-sentence description of the red and blue interval. They then had to determine the

[1]For empirical results with respect to the conceptual adequacy of the interval relations see Knauff (1997).

startpoints and endpoints of a red and a blue interval using mouse clicks. After confirmation of her/his final choices, the subject was told whether her/his choices were correct or false. If they were false, additional information about the correct answer was given. Trials were presented in blocks of all 13 relations in randomized order. The learning criterion for one relation was accomplished if the subject gave correct answers in 3 consecutive blocks of the corresponding relation. The learning phase stopped as soon as the last remaining relation reached the learning criterion. Subjects needed 15 to 30 minutes to accomplish the learning phase.

In the *inference phase* subjects had to solve 12 spatial *4ts-problems* in the *active particular inference paradigm*[2], and the premises $W\, r_1\, X$, $X\, r_2\, Y$, and $Y\, r_3\, Z$ (abbreviated in the following as "$W - X$, $X - Y$, $Y - Z$") were presented in continuous ($W - X$, $X - Y$, $Y - Z$), semi-continuous ($X - Y$, $Y - Z$, $W - X$) and discontinuous ($Y - Z$, $W - X$, $X - Y$) order. They were selected on the basis of our first 4ts-experiment (Rauh & Schlieder, 1997), that the number of correct answers in Rauh & Schlieder (1997) were relatively high and each of the 12 relations were presented in the first premise exactly once. According to the separated-stages paradigm (Potts & Scholz, 1975), premises were presented successively in a self-paced manner, each on an extra screen.

Afterwards (in contrast to Ehrlich & Johnson-Laird, 1982) subjects had to specify **all** three implicit relations $W - Y$, $X - Z$, and $W - Z$, each on an extra screen, by choosing the startpoints and endpoints of the intervals in lightly colored rectangular regions (as they had done in the learning phase). To avoid the effects of presentation order we systematically varied the color of the intervals and the order of conclusions asked for. This made the tasks relatively difficult, since subjects not only had to specify the relations but also to remember the combination of colors in each premise.

The three instances of each of the 12 4ts-problems (12x3=36 tasks) were compiled in different blocks, and there was also one practice block in the beginning consisting of 6 other simple 4ts-problems. The sequence of experimental blocks was counterbalanced across subjects according to a sequentially counterbalanced Latin square. The experiment took approximately 1.5 hours.

Results

All 36 subjects successfully passed the learning phase, and all data collected in the inference phase could be further analyzed. Individual performance showed considerable variation, ranging from 44% to 95% correct answers.

As shown in Figure 2, the results corroborated our prediction in two ways: (1) there was no significant difference of errors between continuous (39.7%) and semi-continuous (40.1%) premise order [$\chi^2_{(1)} = .017$, $p > .898$], but (2) both were significantly easier than the discontinuous order which lead to 50.0% errors on average [$\chi^2_{(1)} = 9.643$, $p < .001$; $\chi^2_{(1)} = 8.864$, $p < .002$.].

[2] In the *active particular inference paradigm* subjects had to find only some conclusions which are compatible with the given premises (Knauff et al., 1995).

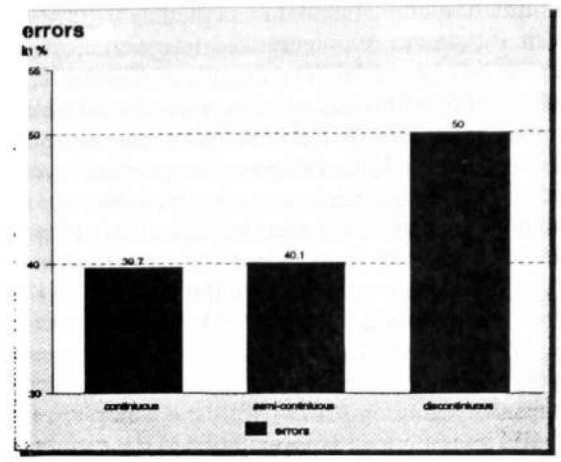

Figure 2: Error rates for continuous, semi-continuous and discontinuous premise order in the 4ts-problems.

Another important finding is reported in Table 1: the data on premise processing times support the assumption of mental model theory that a discontinuous premise order will increase the processing time for the third premise, because information from all premises must be integrated at this point. Reliable differences can be found in the processing times of the third premise between continuous and semi-continuous order [$F(1,35) = 37.61$, $p < .001$], semi-continuous and discontinuous order [$F(1,35) = 40.44$, $p < .001$], and continuous and discontinuous order [$F(1,35) = 74.87$, $p < .001$]. For the second premise the differences between continuous and discontinuous order [$F(1,35) = 17.63$, $p < .001$], and semi-continuous and discontinuous [$F(1,35) = 22.89$, $p < .001$] are significant. All other differences, in particular in the first premise, and the difference between continuous and semi-continuous order in the second premise, are not reliable.

Table 1: Premise processing times for the first, second and third premise in the tasks with continuous, semi-continuous and discontinuous premise order.

	Premise processing time in sec.		
premise order	premise 1	premise 2	premise 3
continuous	13.0	11.2	10.9
semi-cont.	13.6	11.0	14.4
discont.	12.4	13.9	19.5

Discussion

The experiment was conducted to investigate the continuity effect in the spatial domain with the aid of the interval relations. The error rates as well as the premise processing times showed a strong continuity effect. Subjects made more errors in tasks with discontinuous premise order than in continuous and semi-continuous order, and it took more time to process the third premise in the discontinuous condition. These results can be seen as clear evidence for the most important assumption of mental model theory, namely that the information of the premises is integrated into a unified representation, the mental model. The result can than be explained as an effect of the difficulty of integrating the information from the premises. Only in the continuous and semi-continuous order, is it possible to integrate the premises immediately, whereas in the discontinuous order the information from the first and second premise must be kept temporarily separated in working memory (maybe in a language-like propositional form or as separate models) until the third premise is given.

This interpretation is supported by the premise processing times as well, which show that it takes much more time to process the third premise in the discontinuous order. Again, these results are compatible with the assumption that subjects build an integrated representation of the given premises. In fact, the processing time for the third premise in the discontinuous premise order must be longer, because at this point in the model construction process, subjects get the first chance to integrate the first two premises.

Experiment II: Order of Terms Effect

Subjects

24 paid students (12 female, 12 male) of the University of Freiburg, ranging in age from 20 to 33 years.

Method and Procedure

The same interval relations as in Experiment I were used and the computer-aided experiment was again separated into the three phases. The definition phase and the learning phase were conducted as in Experiment I. In the *inference phase*, subjects had to solve 128 *3ts-problems* in the *active particular inference paradigm* (plus 10 practice trails).

Of the 144 possible 3ts-problems, we selected 32 indeterminate tasks (i.e., multiple model problems) that showed the highest degree of preference from our preferred mental models experiment reported in Knauff, Rauh & Schlieder (1995). For each task we constructed "twin" tasks, which use the inverse relation but describe the same spatial relation between the three intervals. For example, the spatial arrangement of *"X lies to the left of Y"* and *"Y lies to the left of Z"* is identical to *"Y lies to the right of X"* and *"Z lies to the right of Y"*.

As shown in Table 2, based on the location of the terms, we constructed tasks of four different types. The complement lines in the table denote the fact that the inverse relation was used. With respect to the terminology of research on syllogistic reasoning the "types" can also be called "figures". In all four types ***Y*** is the middle term, which occurs in both premises of the problem but on different locations. The conclusions connect the two end terms ***X*** and ***Z***, and occur in the premises on different locations as well.

Table 2: 3ts-problems of experiment 2 were constructed in four different types, by changing the term orders and using the inverse interval relations.

type	premise 1	premise 2	possible conclusions
1	$X r_1 Y$	$Y r_2 Z$	$X r_3 Z$ *or* $Z \bar{r}_3 X$
2	$Y \bar{r}_1 X$	$Y r_2 Z$	
3	$X r_1 Y$	$Z \bar{r}_2 Y$	
4	$Y \bar{r}_1 X$	$Z \bar{r}_2 Y$	

In each trial, after reading the premises, subjects first had to decide which interval to use to begin the one-sentence description of the conclusion (in German). This was done by pressing associated keys on the keyboard, namely <B> for *"The blue interval ..."*, <R> for *"The red interval ..."* and <G> for *"The green interval ..."*. Afterwards a new screen was shown, where the second part of the sentence was displayed automatically. This was possible because the middle term could not be used in the conclusion. If, for example, the green interval was the middle term of the task, and the subject had pressed the key <R> initially, the two phrases *"The red interval ... "* and *" ... the blue interval"* are displayed. Between these, a list of all 13 interval relations was displayed (in randomized order), and the subject could choose one of them with the cursor.

Results

As in our previous experiment, all 24 subjects successfully passed the learning phase. Individual performance in this (easier) experiment ranged from 43% to 98% correct answers.

The most important result is concerned with the term orders chosen in the conclusions. Remembering that in general it could be predicted from the results of Johnson-Laird & Bara (1984) that subjects tend to choose the order $X\ r_3\ Z$ (abbreviated as "***X – Z***") more often than the reverse order $Z\ \bar{r}_3\ X$ ("***Z – X***"). This assumption is not supported by our results: 62.8% of all conclusions given by the subjects were in the order ***Z – X***.

As shown in Figure 3, this general bias is modulated by term order in the premises. For the conclusion, the term order ***X – Z*** was chosen for ***X – Y, Y – Z*** (44.3%) more often than for ***Y – X, Y – Z*** (38.7%) ***X– Y, Z – Y*** (39.6%), and ***Y – X, Z – Y*** (31.1%). The difference between ***Y – X, Y – Z*** and ***X– Y, Z – Y*** is not reliable [$\chi^2_{(1)} = 0.134$, $p > .71$], whereas the other differences are significant and show a clear figural bias: The conclusion ***X – Z*** was used more often for the premise order ***X – Y, Y – Z*** than for ***Y – X, Y – Z*** [$\chi^2_{(1)} = 4.959$, $p < .015$], ***X – Y, Z – Y*** [$\chi^2_{(1)} = 3.465$, $p < .035$] and ***Y – X, Z – Y*** [$\chi^2_{(1)} = 28.259$, $p < .001$], and for the premise order ***Y – X, Y– Z*** more often than for ***Y – X, Z – Y*** [$\chi^2_{(1)} = 9.640$, $p < .001$]. The difference between ***X – Y, Z – Y*** and ***Y – X, Z – Y*** is also reliable [$\chi^2_{(1)} = 12.036$, $p < .001$].

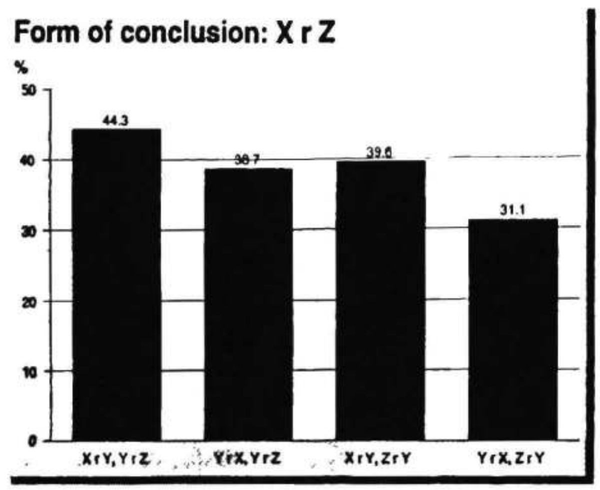

Figure 3: The effect of term order in the premises on the form of conclusions. The figure shows the distribution of X r Z conclusions [in %] for 32 problems in each of the four types.

As mentioned above, in an earlier experiment we have found preferred mental models for problems with multiple solutions (Knauff & al., 1995). We now look at the solutions of our 32 indeterminate problems and compare the conclusions with these preferences. Two results are important: (1) the preferences we found in the experiment were independent from the order of terms; only in one of the 32 tasks did the preferences differ in the four term orders. (2) in all cases we found strong preferences, the majority of which (24 out of 32, or 75%) were identical to those found in our previous investigation (Knauff & al. 1995).

Discussion

The experiment was aimed more at proving the existence of figural effects well-known from other domains of reasoning. We found (i) contrary to the results of Johnson-Laird and Bara (1984) no general bias towards $X-Z$ conclusions, and (ii) in accordance with Johnson-Laird and Bara a figural effect, i.e. the figure $X-Y$, $Y-Z$ favored $X-Z$ conclusions, whereas the figure $Y-X$, $Z-Y$ favored $Z-X$ conclusions.

The first contradictory result of the $Z-X$ preference could be attributed either to the spatial domain or to the property of the used relation in the Johnson-Laird and Bara study; they used the relation "is related to" denoting kinship, a relation that has the property of symmetry in contrast to the normally used material in relational reasoning studies ("better than", "taller than", ...) and also in contrast to our qualitative relations that do not have the property of symmetry. The effect of abstract properties of relations like symmetry, asymmetry, or anti-symmetry, and the effect of the domain of relations (spatial v. non-spatial) on the preference of $X-Z$ conclusions has to be determined in future experiments. However, the overall effect of a $Z-X$ preference can be explained by a cognitive process that inspects the mental model by means of a spatial focus and is sensitive with respect to the outcome of the model construction. This explanation is sketched in the general discussion below.

The next result was that the type of preferred mental model seems to be independent of the order of terms in the premises, since in nearly all of the *3ts-problems* the same relation between the end-terms was chosen for all four figures per task. In addition, the stability of mental model preferences determined in the study of Knauff et al. (1995) was not perfect, although within the range of variability found in a replication of the latter experiment in Kuß, Rauh and Strube (1996).

General Discussion

We reported two experiments investigating the "continuity effect" and the "figural effect" in spatial relational inference tasks. Taken together, our findings support an account of the inference process following mental model theory and pose some problems for a syntactically oriented, rule-based theories.

In the first experiment we found evidence for premise integration and showed that discontinuous premise order is much harder than semi-continuous and continuous order. The result is easy to explain on the basis of the difficulty of integrating the information of the premises. This explanation is clearly supported by the premise processing times.

On the other hand, the results of the second experiment leave us with some open questions. The main idea of the experiment was to investigate another factor possibly effecting spatial relational inference in a similar fashion to the way that premise order does. The results of this experiment were surprising, particularly in one point. Contrary to the results of Johnson-Laird and Bara (1984), we found no general bias towards $X-Z$ conclusions. In fact, subjects tend to generate conclusions of the form $Z-X$. At first glance, this result seems to be counterintuitive, but may have a plausible explanation: The overall effect of a $Z-X$ preference can be explained - in agreement with the assumptions of mental model theory - by a cognitive process that inspects the mental model by means of a spatial focus (e.g., De Vooght & Vandierendonck, in press). According to the computational account given for the construction of spatial mental models by (Schlieder, 1995), after constructing the mental model the focus should be positioned on the last end-term of the *3ts-problem*, namely Z. If this is the starting point of the scanning process, it is plausible that the first term in the conclusion is Z and not X, because for $X-Z$ conclusions the focus must be shifted back to the term X before the scanning process starts. In contrast, $Z-X$ conclusions do not need time consuming re-focussing, which explains why our subjects preferred those conclusions in our second experiment. The observed preferences results directly from the process of model construction as detailed in Schlieder (1995). The alternative computational account presented by Berendt (1996), however, leaves it an open question how the present results can be modeled.

An even more important result of the second experiment is that the term order in the premises does not effect the model preferences of the subjects. In 31 of the 32 problems the same preferred mental model was chosen for all four types of term order.

Taken together, both experiments give us important hints concerning the processes of model construction and model inspection. The model construction process seems to be sensitive to the order of premises and widely independent of term order. The importance of term order emerges as son as the model is inspected, because this process seems to be sensitive to the position of the focus, resulting from the model construction process. However, further work is needed to integrate model inspection in the existing computational accounts of how spatial mental models are constructed.

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