What and How Schema Networks Are Acquired During the Learning of Line Graphs: Modelling Using Representational Systems Theory

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Abstract

This paper addresses a gap in our understanding of cognition with external representations: what memory structures are acquired and how they change during learning? The focus is on line graphs. We adopt Representation Interpretive Structure Theory (RIST) and its modelling notation (RISN) as an approach to answer those questions. RIST is a schema theoretic account and RISN operationalizes its assumptions. Models for stages in the gradual acquisition of growing interpretive sophistication are built, from basic precursor components to advanced interpretations. Learning mechanisms are proposed to explain the transition between the stages. During learning, the memory structures undergo localized incremental changes and more global radical restructuring of the networks.

Keywords: External representations, schema theory, interpretation, memory structures, Line graphs.

Introduction

Many accounts of how humans perceive, interpret and use external representations, ERs, have been proposed (e.g., Larkin & Simon, 1987; Zhang & Norman, 1994a, 1994b; Kirsh, 2010; Shimojima, 2015); for a review, see Hegarty (2011). However, our understanding of the memory structures acquired during the *learning* of external representations is less mature. What informational structures are built in memory as we learn about an ER? How do those structures change with growing experience of using ERs? Do large scale changes happen to the structures or is their development largely incremental?

Our focus is on *Cartesian coordinate system line graphs* as a representative class of ERs. Line graphs (for short) provide a good case for the overall research question, for four reasons. First, they are important and widely deployed class of external representation. Second, as their design comes in many variants and levels of complexity, they provide a rich pool of examples for investigation. Third, studies of how we read and interpret line graphs have been conducted (e.g., Kosslyn, 1989; Pinker, 1990; Tabachneck-Schijf, Leonardo & Simon, 1997; Carpenter & Shah, 1998; Peebles & Cheng, 2003). They describe phenomena that need to be explained by an account of memory structures of line graphs. Fourth, studies of the misconceptions of children learning about line graphs have been conducted (Kerslake, 1977; Janvier, 1981; Barclay, 1986; Clement, 1985). They also describe phenomena that need to be explained by an account of memory structures.

However, although some of the studies provide accounts of possible memory structures of mental representations of line graphs, both descriptively (Pinker, 1990) and as computational models (Tabachneck-Schijf, Leonardo & Simon, 1997; Peebles & Cheng, 2003), an account of what memory structures are acquired and how they change with learning is yet to be provided. And especially, one that might generalize to ERs more widely.

Goals

The specific goals of this paper are twofold. The primary goal is to theoretically examine *what* schemas are used for the interpretation of line graphs and they *how* they change with learning. Our method is to build models for diverse cases concerning the interpretation of line graphs covering recognized examples of phenomena from multiple perspectives. The overall goal is to provide an account that covers all these phenomena within a single set of closely related models. Taken together, the models will argue for the validity of our overall account through the coherence of the memory structures that are proposed.

For these models of memory structures we adopt *Representational Interpretive Structure Theory*, RIST (Cheng, 2020, Cheng et al., 2022, Stockdill et al., 2022). RIST is a schema-based account of the memory structures that we deploy when using ERs. RIST is operationalized in a modelling notation, RISN. A web browser-based editor, RISE, is provided for building models in RISN that conform to RIST's theoretical prescriptions.

The secondary goal of this paper is to further test the utility and validity of RIST and RISN in terms of whether models for a wide range of line graph phenomena can be produced within a coherent interpretive framework. Although RISN models of line graphs have previously been proposed (Cheng et al., 2024b), the novel challenge of this paper is whether RIST can be applied to *changes* in memory structures that occur during learning; so, it moves the theoretical explanations beyond snapshots of fixed interpretations.

Method

For this cognitive modelling study, we will follow and extend the approach of Young and O'Shea (1981), which was applied to the explanation of children's multi-digit subtraction errors. First, they chose a large dataset of subtraction errors, which were classified into seven types of errors. Second, using a production system, they build a base-competence model of correct subtraction problem solving. Third, by either ablating the goal hierarchy of the model or by augmenting it with short-cut rules, they explained each of the seven errors.

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Their model variants successfully encompass most of the subtraction errors.

Here, we adopt a base-competence model of line graph interpretation in RISN provided by Cheng et al. (2022). These models are presented in the next section, simultaneously with the introduction of RIST and RISN. Elaborations and extensions of the base-competence model are then developed in the Results section. Conceptualizations of line graphs that are precursors or successors to the base-competence model are considered across various perspectives. These include: the misconceptions of children who are just beginning to learn about line graphs (Kerslake, 1977; Janvier, 1981; Barclay, 1986; Clement, 1985); learning about the basic components that underpin all line graphs (e.g., axes); use of technically complex graphs (Cheng, 2020); sophisticated inferences supported by extended, alternative, and specialist formats of line graphs. The Discussion considers the extent to which the modelling provides a successful account of the nature and development of memory structures in the interpretation of line graphs.

RIST and RISN, and base interpretation

Cheng (2020), Cheng et al. (2022), Stockdill at al. (2022) introduced RIST, RISN and RISE. To summarize RIST and RISN, consider a model of a typical interpretation of a simple Cartesian coordinate line graph by an individual who is competent in their use. Fig. 1A shows a sample line graph with variables Vx and Vy, with six explicit datapoints, and with lines to interpolate values between the data points. It is assumed the user takes a *functional* interpretation of the graph in which Vx is an independent variable whose manipulation influences Vy as a dependent variable. Each value of Vx has only one value of Vy, but one Vy may map to multiple values of Vx. The labels in green are not part of the line graph but are cross references to the model. Fig. 1B is the RISN model of this line graph.

The key theoretical claim of RIST is that the primary function of the schemas that we possess for interpretating ERs closely associate domain concepts with graphical objects in the ER. RIST proposes four classes of schemas - they have icons with specific shapes in RISN: *Representation* - lozenge [e.g., Fig. 1B, coordinate 3A]; *R-scheme* - rectangle [e.g., 3B]; *R-dimension* - trapezium [e.g., 1D], *R-symbol* rounded rectangle [e.g., 1E]. All the schemas have a slot for the domain concept at the top of each icon, and a slot for the graphic object at the bottom of each icon (other slots for the schemas are not shown). The labels for graphic objects use this convention: plain text for labels in the graphic (e.g., the green text in Fig. 1B); descriptions of graphic objects in curly brackets, e.g. {x-axis}; text in quotes for text that literally appears in the graph, e.g., “Vx”.

To be quite explicit, Cheng and colleagues claim that the four schemes are the internal mental representations that are processed mentally whenever we use a graphical external representation. The content of the upper slot in a schema’s icon is a concept about the domain of interest in the declarative memory of the user. The content of the lower slot is a pointer (or pointers) to some content in visual memory that originate from perceptions of a graphical object in the external representation.

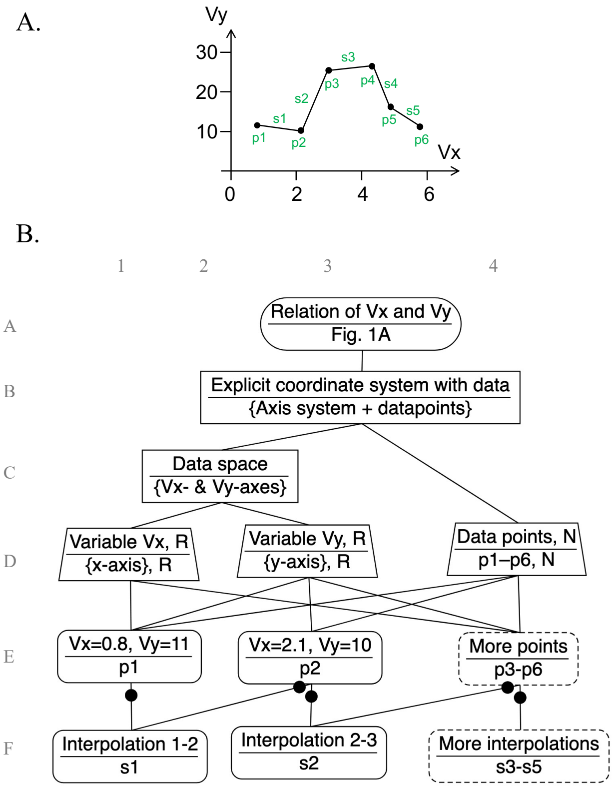

Figure 1. (A) A simple line graph; (B) The RISN model of the interpretation of (A).

Consider each class of schema in turn. A Representation schema associates (i) a cluster of coordinated ideas about the domain with (ii) an overall external representation. In our example, a functional relation between Vx and Vy represented by the line graph [Fig. 1B, 3A].

A R-symbol schema associates (i) a basic fixed value concept with (ii) a discrete icon or item of text in the external representation. An example is a datapoint with a pair of values and a dot icon in the graph [3E]. A class R-symbol has a dashed outline and stands for a collection of individual R-symbols [e.g., 4E]. Fig. 1B treats points p1 and p2, and line segments s1 and s2, individually whereas all the rest are considered as a group of points [4E] and as a group of line segments [4F].

A R-dimension schema associates (i) a concept that ranges over alternative values, such as a variable or a set with (ii) a graphic object that performs some form of grouping function, such as an axis, a container, or shared styling. Examples are the two axes [1D, 3D] and the array of datapoints [4D]. The concept and graphic objects of R-dimensions can be characterized in terms of their quantity scale, which may be nominal, ordinal, interval or ratio. The properties of concepts and graphic objects in R-dimensions is indicated by the letter following to the right of each line in the trapezoidal icons (one of N, O, I, R). All the quantities for the R-dimensions for the two variables are ratio scales [1D, 3D] and the two quantity scales for the concept and graphic object of the datapoint R-dimension are nominal [4D]. RIST/RISN pays particular attention to the nature of quantity scales, because of their influence on the cognitive load of using an external representation (Cheng et al., 2024).

A R-scheme schema is a combination of R-symbols and R-dimensions, and occasionally sub-R-schemes or sub-Representations. A R-scheme associates (i) a complex compound of concepts with (ii) an assembly of graphic objects and the conditions applying to their relations. There are two R-schemes in our example. One is the *data space* formed by the two variables, which in standard line graphs includes the concept that the axes are arranged orthogonally [2C]. The other R-scheme is the *explicit coordinate system* [3B]. It consists of the data space R-scheme [2C] plus a R-dimension for a collection of data points [4D] and it also includes the concept that there is a functional relation between *Vx* and *Vy*.

The data space and explicit coordinate system are examples of *idioms* (Stockdill et al., 2022). Idioms are prototypical RISN structures that can be found in many domains and across diverse types of representations. For example, a RISN model for a polar coordinate axis system would have the same idioms (and overall structure) as Fig. 1B, but the graphic objects of the R-dimensions differ (i.e., angle and radial distance). Explicit coordinate systems are distinct from *implicit* coordinate systems, as will be explained below. Coordinate systems with sets of axes are an example of a larger class of *triangulational coordinate systems* that are in turn one subclass of *indexing systems* more generally (Cheng et al., 2024).

RIST requires RISN models to be hierarchical structures in which information from schemas higher in the network is inherited by lower schemas. The links between icons in RISN models represent these hierarchical inheritance relations. In addition to the generic hierarchy links, RISN includes *anchor links*, which have a dot near the start of their lines [1E-F]. Anchor links place a restriction on the scope of the child schema to indicate that its concept and graphic object are to be interpreted locally in relation to the parent schema. In Fig 1A, each line running between pairs of datapoints can be interpreted as an interpolation line that is only applicable to positions between the datapoints; hence, anchor links run between the R-symbols from those datapoints in Fig. 1B [e.g., 1E-F, 3E-1F].

One may draw an analogy between chemical theory and RIST, in order to appreciate the intended purpose of RISN models. In chemistry the properties of a chemical are explained by the structure of a molecule and the atoms it contains. Likewise, in RIST the properties of a representation are explain by the structure of its RISN model and the schemas that it contains. In chemistry elements can only be combined in certain configurations and, in a similar way, only certain of schemas can be meaningfully connected. RIST provides rules for the construction of valid networks (Cheng et al., 2022). For easy of modelling, the RISE browser-based modelling tool has the theoretical constraints of RIST built in (Stockdill et al., 2022).

Cheng & colleagues do not claim that a single Platonically correct RISN model exists for a given external representation. Rather RISN enables an analyst (e.g., a teacher) to build alternative schema networks of the same external representation; for example, to model interpretations of users with differing levels of familiarity of a given representation. The analyst can also build alternative interpretations derived from the same individual under different task goals (Cheng et al., 2022) or to model that individual's learning changes.

We can apply a *sensibility test* to make an initial assessment of the reasonableness of a given model by reading the schemas and links from the node to the leaves of the network, or vice versa. If the reading provides a succinct meaningful explanatory description of how the external representation encodes the domain knowledge, then we may claim that the network captures the interpretation adequately. For example, this is a reading of Fig. 1B. Fig. 1A is a line graph of the relation between *Vx* and *Vy* [A3]. It deploys a coordinate system to display data [3B]. The data space [2C] is defined by two axes for *Vx* and *Vy* [1D, 3D], within which datapoints are plotted [4D]. Each datapoint has specific pair of values [1E-4E]. The line segments between successive pairs of datapoints can be used to interpolate values [1F-4F].

Cheng (2020) demonstrates the scope of RISE by building RISN models of ERs with diverse formats (e.g., equations, graphs). Cheng et al. (2022) show that alternative interpretations from different analysts of the same ER can be captured in RISN models. Cheng et al. (2024b) used RISN models to provide a novel cognitive systematization of classes of indexing systems. Cheng et al. (2024a) provide criteria for assessing the cognitive demands of ERs and show that RISN models successfully predict the outcomes of empirical tests when looking at the relative efficacy of competing ERs. That completes the overview of RIST and RISN.

RISN learning mechanisms

Here we propose five *RISN learning mechanisms* that change memory structures as a student's understanding increases.

C1. *Slot modifications* change the contents of the slots of schemas; for instance from a default value for a class of ER into a value for a special instance of that class.

C2. *Network chunking* occurs when some subnetwork is replaced by a single schema.

C3. *Local network updating* revises the structure of a RISN network locally so that either (a) a wider variety of cases is encompassed or (b) to eliminate conceptual redundancies by simplifying the network.

C4. *Idiom acquisition* produces a new idiom – prototypical configuration of schemas – when a learner recognizes that the same network structure occurs across different ERs and domains.

C5. *Global network revision* is wholesale change to the overall structure of a network when a radical novel interpretation of a representation is adopted. This will likely occur incrementally with both the original and

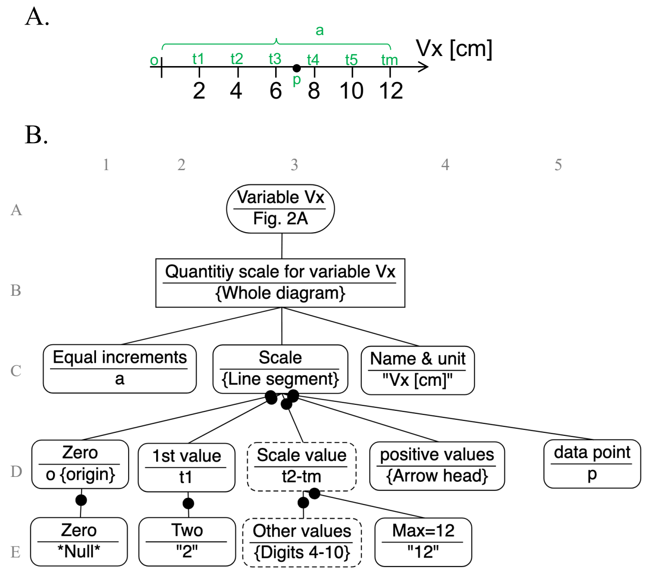

Figure 2. (A) An axis scale; (B) its RISN model.

new interpretation being simultaneous entertained for some time.

Will this set of mechanisms provide reasonable explanations of the learning of representations, which are both necessary and sufficient, for the dual objectives of this paper? To reiterate, they are: (i) to examine how the memory structures of users of line graphs change with learning using RIST; (ii) to test the validity of RIST by applying it to a range of line graph related phenomena.

Results

Learning about axes & coordinate systems

The base-competence model was presented in the previous section (Fig. 1). A prerequisite for that interpretation is a knowledge of axes and coordinate systems. A student introduced to line graphs for the first time must first learn about axes. What concepts must they acquire as they are given the task of reproducing Fig. 2A, which involves drawing an arrow, labelling it, adding tick marks, and numbering the ticks? Fig. 2B is a possible RISN model of the diverse information that has be mentally encoded and integrated by a student, including: that there is a target variable being represented, (e.g., *Vx*); it has a unit (e.g., cm); it has a quantity scale with various properties, such as its linearity and its range; there is a mapping between values of the variable and the scale.

A student's detailed knowledge that an axis is a numerical quantity scale is captured by a R-Scheme [Fig. 2B, 3B] with three further levels of schemas. On the first level an association is made between the concept of a scale and the drawn line segment, in a R-Symbol [3C]. Another R-symbol then identify the name with the written label [4C]. On the second level, the existence of scale values and their representation by tick-mark place-holders is encoded by a series of R-symbols [1D-3D], which are anchored to the scale R-symbol. The first tick is the start of the scale and the origin of the axis [1D], it has a larger tick mark than the rest. In the model, only the first of the series of ticks is shown as an individual R-symbol [2D] and the rest are encoded as a class R-symbol [3D], for brevity. The student's idea that the scale spans positive numbers depicted by a right pointing arrowhead is captured by an R-symbol [4D]. The student can see that the spacing between the ticks is the same, so a R-symbol in the level above encodes this idea about equal increments [1C].

The student's understanding of scale values is captured by the R-symbols at bottom level of the network [1E-4E], each anchored to a R-symbol above. The first value of the scale represents the concept *two* and has a "2" digit as its graphic object [2E]. The origin has a scale value of zero, but as no "0" digit is actually written, the R-symbol schema has *Null* code in the graphic object slot [1E]. The scale terminates with a R-symbol for the maximum value [4E], which is anchored to the class R-symbol above [3D]. That also has a child class R-symbol to encode the rest of the values [3E]. The student's knowledge of the association between the axis graphic object and the target variable is captured by the apex Representation schema [3A].

To acquire the network in Fig. 1B a student will have used by a process of (C1) slot modifications to complete schemas and (C3) local network updating to successively setup associations by introducing new schemas.

Other students might acquire different RISN network structures for this axis. For instance, one might encode each pair of *value placeholder–tick mark* and *value–digit* R-symbols as a single combined R-symbol, which would be an example of (C2) network chunking. That student would conceptualize them as one schema, if they were instructed to write each tick and digit simultaneously. Here is another example. Local network updating (C3) might produce two sub-R-symbols of the name-and-unit R-symbol [4C], one for the name and one for the unit, because a student is told that the variable and its unit are distinct concepts.

Now, once a student has understood axes, they can use them to represent values of variables by spatially aligning datapoints with the axis. For instance, in Fig. 2A, the student has been instructed to draw a dot on the axis to represent the concept *Vx*=7 cm, which creates a new R-symbol [5D] anchored to the scale R-symbol in the network in Fig. 2B. In a full line graph, a student may be asked to trace, or even draw, lines from data points in the space to perpendicularly intersect the axis. Such additions can be modelled by adding R-symbols to Fig. 1B for such construction lines. These are examples of C3 local network updating by elaboration.

With growing familiarity of axes and how to place data points to encode values, a student will begin to treat the contents of Fig. 2B as a single integrated conceptual entity, such that the processing of all the R-symbols grows ever more automatic and do not need to be explicitly examined in working memory. This is (C2) network chunking that replaces the network with an R-dimension. With exposure to orthogonal axes, the student's network will evolve into a structure for a complete interpretation of a Cartesian coordinate system, as in Fig 1B, which is an example of (C4) idiom acquisition.

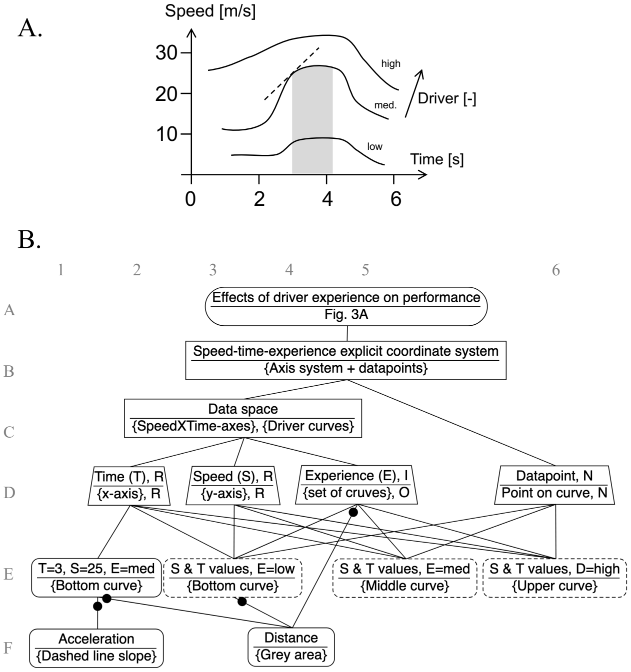

Figure 3. (A) A three-variable graph with some advanced interpretations of local features; (B) its RISN model.

More sophisticated interpretations

With growing experience of using line graphs for challenging tasks, students will continue to elaborate their networks and discover new interpretations. Imagine a student given Fig. 3A as a follow up to Fig. 1A. The plot is a *speed–time* (*S-T*) graph for a specific part of a racing track. It incorporates a continuous third variable for driver experience (*E*), that is represented by low, medium and high curves. No dots are marked in the graph, so the student must imagine points in the space as data values.

Like the network for Fig. 2B, the student's network for this new interpretation, Fig. 3B, also comprises a data space idiom [3C] and an explicit coordinate system idiom [4B]. However, the data space idiom now has a third experience R-dimension [5D], with an interval conceptual scale but an ordinal graphic scale as three discrete values are represented. Just one data point is shown as an individual R-symbol [1E], the rest are represented by class R-symbols [3E-6E].

In Fig. 3A the instructor has introduced new concepts for the student's analysis of driver performance. The first is acceleration, which is the rate of change of speed. At first, just a single case is considered at T=3 s and S=25 m/s. It is represented by the gradient of dashed line that is a tangent to a point on the medium experience curve. This is encoded as a R-symbol [Fig. 3B, 1F] anchored to the individual R-symbol for that datapoint. As more examples of acceleration or distance are introduced, the student will add more acceleration R-symbols to the network. In turn, this set of R-symbols will be generalized into a new R-dimension, as a network chunking process (C2), analogous to the creation of the R-dimension for the variable *Vx* and the axis in Fig. 1B. The new acceleration R-dimension will be a new child of the data space R-scheme [3C]. Similarly, the instructor may show how distances covered in a given time frame are represented in the graph, by drawing the grey area under a curve in Fig. 3A. This produces another R-symbol in Fig. 3B anchored under two R-symbols that define the boundaries of the area [4F]. Again, as more of these R-symbols are examined, it is hypothesized that the student will eventually generalize them into an R-dimension for distance belonging to the data-space R-scheme. All this is an example of one way in which more sophisticated interpretations of lines graphs may arise through the accretion of new R-dimensions for concepts whose meanings are dependent on the primary variables of the domain (speed and distance in Fig. 3).

An alternative interpretation to function line graphs are *parametric* line graphs. In a parametric interpretation multiple y-axis values may be mapped to one x-axis value. *Phase plots* are parametric graphs that show cyclical processes over time. STEM students are often required to use parametric line graphs, but an informal review reveals that texts do not to include detailed explanations of how they work. Peebles and Cheng (2003) show that single line parametric plots are superior empirically to informationally equivalent double line function graphs. Cheng et al. (2024a) produced RISN models for both types of graphs and showed that the superiority of parametric graphs is due to its simpler coordinate system structure. Thus, students' who know how function graphs work can easily learn to use parametric graphs by a (C3) process of local network updating that reduces the number of schemas and the number of links in the network.

Misinterpretations of line graphs

So far, we have considered valid interpretations of line graphs. Unsurprisingly, children possess diverse misconceptions early in their learning of line graphs (Kerslake, 1977; Janvier, 1981; Barclay, 1986; Clement, 1985). Like Young & O'Shea's (1981) modelling of observed errors in multidigit subtraction as ablations of and augmentations to a base-competence model, can RISN models adequately explain the reported diversity of misconceptions in line graphs in terms of variations of the models of correct line graph interpretation presented so far?

Kerslake (1977) devised a rich set of questions to probe children's misconceptions. Nine of the questions were described, which covered ideas about: continuity - whether it is reasonable to interpolate between data points; discrete versus continuous variables; valid line shapes in function graphs; impacts of changes of scales with the same data; how equations for straight lines map onto position and orientation of the graph line; scales that include negative numbers; comparison of gradients; coordinate system based on radiating circles. Examples of children's responses to each of the questions were reported (Kerslake, 1977).

Models for these responses were created. The correct responses are consistent with the base-componence model. Plausible RISN models for all the incorrect responses – excluding those that Kerslake (1977) considers bizarre – were also built. For example, alternative misconceptions about whether interpolation between datapoints and the continuity of values is modelled, by changing the quantity scales assigned to the concept or the graphic object slots of R-dimensions for the variables and axes. This misconception could simply be overcome by the (C1) slot modification learning mechanism. The model of a student who failures to abide the prohibition of multiple x-values of a functional interpretation has a network that shares the common features of the function and parametric models of Cheng et al. (2024) but fails to differentiate them. Correcting this misconception would require (C3) local network updating to capture the x-axis to y-axis mapping rules.

Clement (1985) provides a partial taxonomy of line graph misconceptions, which proposes two main classes: (a) the selection of an incorrect graph feature for a concept; and (b) the *graph as a picture misconception*. Clements's (1985) incorrect selection misconception is trivial to model as (C1) slot modifications.

The idea of misconceptions arising from the treatment of a line graph as a picture was first proposed by Janvier (1981). For example, one of Janvier's test items showed a wavey graph of the speed against time of a racing car and plan views of alternative racetracks to match to the graph. Children often selected the racetrack with the same visual appearance as the graph. Garcia Garcia & Cox's (2010) found that treating the data display as a picture was not unique to line graphs, but also occurred with bar and pie charts, for instance.

Our model of the graph as picture misconception differs from the previous models discussed so far, because it requires three sub-Representation schemas just below the root node of the RISN network. Each has its own sub-network of schemas. The first subnetwork encodes some or all of the basic-competence model, such as Fig 1B. The completeness of the model reflects the child's developing understanding of line graphs. The second subnetwork encodes the child's incorrect literal interpretation of the line, and perhaps also the x-axis, as a picture. For the wavey line graph in Janvier's (1981) task, the model is a hierarchy of R-symbols for nested parts of the curve, including the peaks and troughs, and then their sides. The third subnetwork is a correct literal interpretation of the shape of the track, with its straights and corners. In outline, the process of overcoming the misconception (over and above acquiring the base-competence model) is a process of (C3) local network updating, in which links to the track network from the graph as picture network are abandoned and replaced with links to the basic-competence network. Over time, this cumulatively produces a (C5) global network revision, with the eventual abandonment of the picture network.

In sum, it appears that RISN models for most misconceptions that students have of line graphs can be accommodated, which in turn suggest what RISN mechanisms might be applied to rectify the misconceptions with learning.

Discussion

To address the primary goal we set, we initially adopted a base-competence model of interpretation of lines graphs (Fig. 1). Adaptations to that model showed that RISN can model more advanced interpretations and non-stand conceptions, such as a parametric graph (Fig. 3). How underpinning precursors of graph comprehension are learnt was examined, such as the encoding of variables in axes, (Fig. 2). The facility of RISN to model misconceptions was also demonstrated, with their diversity being accommodated using the given schemas, connections and idioms of RISN. The proposed RISN learning mechanisms provided underpinned the explanations of how new networks might be acquired or restructured. It is clear from the models that learning must combine local incremental changes within and between schemas, but also that wholesale radical shifts in RISN networks may be required of learning.

Previous work with RIST modelled static correct interpretations of ERs (Cheng, 2020; Cheng at al., 2021; Stockdill et al., 2022; Cheng et al., 2024b) and the comparison of the efficacy of ERs (Cheng et al., 2024a). Here, RIST has been extended to the modelling of learning and misconceptions. Although prior work on the learning of line graphs has been conducted, accounts of what memory structures are acquired and how they develop appear to be absent from the literature. The application of RISN to the learning of line graphs is an initial step towards rectifying this lacuna.

Like any language for modelling complex cognition, RISN provides much flexibility to create different models for the same phenomenon. However, our experience of using RISN leads us to concur with Cheng et al. (2021) that modelling with RISN is not arbitrary. The theoretical constraints of RIST – limited variety of four schemas, two types of connection, the prescription of an inheritance hierarchy, and joint encoding of concepts and graphical objects in the schemas – limit the space of plausible models that can be built. Nevertheless, we built for all the cases examined, which supports a positive response to our second goal that concerns the adequacy of RIST and RISN. However, the detailed structure of the models did depend on the authors' presumptions about how users would interpret the ERs. So, it is clear that future work should develop methods to constrain the construction of RISN models using empirical data, such verbal protocols.

Finally, we can mention a possible application of RISN, and the proposed RISN learning mechanisms, to the systematic design of instructional sequences for learning about line graphs. For instance, Fig. 2, Fig. 1 and then Fig. 3 are waypoints on a journey from no comprehension to a relatively advanced understanding of line graphs. The RISN learning mechanisms suggest how transitions from one level of understanding might be built into teaching material.

Acknowledgments

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