A Quantum Model of Arousal and Yerkes Dodson Law

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Abstract

We present the Oscillating Field Perturbation (OFP) model, a quantum model that provides a quantitative account of the Yerkes-Dodson law concerning the relationship between arousal and performance. Inspired by neural models, OFP conceptualizes cognitive control as an oscillating field that perturbs a quantum system, and represents arousal as the "gain" induced by this field. By integrating OFP with the Multiple Particle Multiple Well (MPMW) framework, we demonstrate that OFP successfully explains how the shape of the Yerkes-Dodson law varies with task difficulty and familiarity, consistent with empirical findings. To the best of the authors' knowledge, OFP is the first model to provide a unified account of the empirical variations in the shape of the Yerkes-Dodson law.

Keywords: computational modeling; cognitive control; quantum cognition; arousal;

Introduction

Cognitive control is a fundamental process that guides attention, regulates emotions, and resolves conflicts between competing priorities (Posner & Snyder, 1975; Hammond & Summers, 1972; Ochsner & Gross, 2005; Cohen, Dunbar, & McClelland, 1990; Botvinick, Braver, Barch, Carter, & Cohen, 2001). A phenomena closely related to cognitive control yet underexplored is arousal. Arosual refers to the level of physiological and psychological activation and alertness (Pribram & McGuinness, 1975; Thayer, 1990). For instance, the surge of anxiety before a public presentation reflects hyperarousal (excessively high arousal), whereas the drowsiness experienced during a monotonous task reflects hypoarousal (insufficient arousal). Numerous neuroscientific and behavioral findings suggest that arousal plays an important role in various cognitive processes (Bogacz, Brown, Moehlis, Holmes, & Cohen, 2006; Sohn et al., 2015; Wichary, Mata, & Rieskamp, 2016; Hulsey, Zumwalt, Mazzucato, McCormick, & Jaramillo, 2024). However, the quantitative relationship between arousal and cognitive control remains poorly understood (Cohen, 2017).

The Yerkes-Dodson Law (see Figure 1a) is a foundational theory describing how arousal influences cognitive performance. It proposes that performance generally follows an inverted U-shaped curve, peaking at moderate arousal levels and declining at both extremes (Yerkes, Dodson, et al., 1908). Decades of research have provided empirical support for this relationship across perception, memory, and decision-making tasks (Easterbrook, 1959; Kahneman, 1973; Landers, Arent, & Lutz, 2001; Aston-Jones & Cohen, 2005; Anderson, 1994).

Various models of cognitive function have taken account of arousal and the Yerkes-Dodson Law (Usher & McClelland, 2001; Easterbrook, 1959; Kahneman, 1973; Mather & Sutherland, 2011). However, these models have not generally provided a explicit mechanistic account of how it arises, nor how task difficulty and familiarity influence the shape of the arousal-performance relationship. One line of neural network models, the Adaptive Gain Theory, have been developed to provide an account of how neuromodulatory brain mechanisms may regulate arousal and how this may explain the Yerkes Dodson Law (Servan-Schreiber, Printz, & Cohen, 1990; Aston-Jones, Rajkowski, & Cohen, 1999; Aston-Jones & Cohen, 2005). Specifically, the model proposes that the locus coeruleus–norepinephrine system (LC) dynamically regulates "cortical gain," optimizing performance at moderate arousal while causing performance degradation at both low and high arousal levels, thus reproducing the Yerkes-Dodson Law. While this model offers a neuroscientific account of the Yerkes-Dodson Law, it has not been directly integrated with other cognitive processes, and in particular mechanisms thought to be responsible for cognitive control.

A recent line of research, quantum cognitive modeling (Huang et al., 2025; Pothos & Busemeyer, 2022), offers a novel approach to addressing this gap (Rosendahl, Bizyaeva, & Cohen, 2020; Rosendahl & Cohen, 2022; Vol, 2012). Inspired by neural insights, quantum models of cognitive control provide several advantages. First, they inherently capture the stochastic nature of neural systems without requiring additional stochastic variables (Rosendahl et al., 2020). Second, they offer a probabilistic framework for representing energy, aligning with how arousal is modeled in adaptive gain control theory (Rosendahl & Cohen, 2022).

The Multiple Particle Multiple Well (MPMW) model, proposed by Rosendahl and Cohen (2022), is the most advanced quantum approach to date for integrating arousal with the dynamics of human decision-making and cognitive control. It models arousal as the energy eigenstates of the Schrödinger equation, where dynamic changes in these eigenstates reflect how arousal modulates decision-making performance. As highlighted by Rosendahl and Cohen (2022), the model successfully explains various perceptual decision tasks under cognitive control, such as two-alternative forced choice tasks, and provides a framework for incorporating arousal into multi-alternative decision-making.

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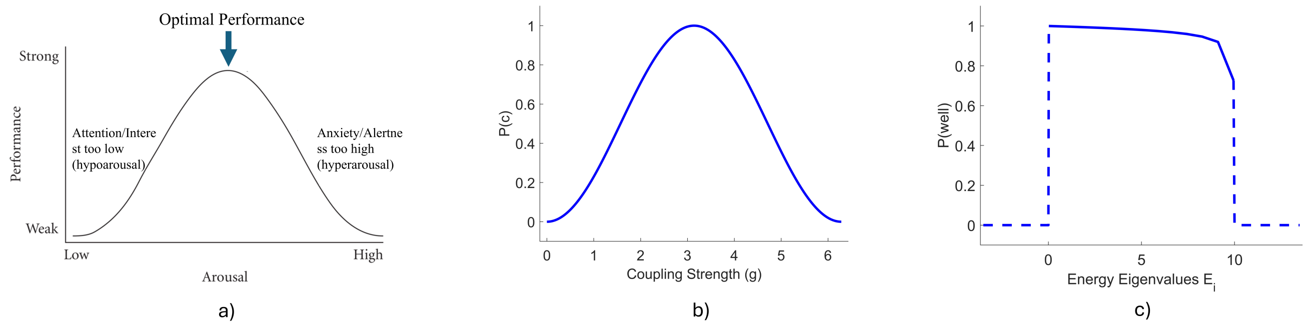

Figure 1: Yerkes Dodson law and model predictions. a) **Original Hebbian version of the Yerkes Dodson law.** Performance is suboptimal at both low and high arousal levels. Low arousal (hypoarousal) leads to distraction and disinterest, while high arousal (hyperarousal) causes anxiety and excessive alertness. b) **OFP's prediction of the Yerkes Dodson law in a two-level quantum system.** The coupling strength represents arousal, and $P(c)$ corresponds to the probability of successful cognitive operations (performance). c) **MPMW's Prediction of the Yerkes-Dodson Law.** Discontinuities occur at $E_i = 0$ and $E_i = 10$, where the probability of being in the well drops sharply to zero. Here, $P(\text{well})$ represents the probability of detecting particles in a potential well, which also corresponds to the choice probability of the correct option (performance).

Despite its strengths, the MPMW model has notable limitations. It accounts for the "sharp-drop" version of the Yerkes-Dodson Law (see Figure 1c) but fails to reproduce other variations of the law. Besides, while the MPMW model links cognitive efficacy to performance, it does not explain how these relationships give rise to variations of the Yerkes-Dodson curve, particularly in relation to task familiarity and difficulty.

In this work, we introduce the Oscillating Field Perturbation (OFP) model, an extension of the MPMW framework that addresses its limitations, providing a unified account of the different forms of the Yerkes-Dodson law. Inspired by the Adaptive Gain Theory (Aston-Jones et al., 1999), as well as widespread observations of neural oscillatory activity (Totah, Neves, Panzeri, Logothetis, & Eschenko, 2018; Usher, Cohen, Servan-Schreiber, Rajkowski, & Aston-Jones, 1999), the model represents cognitive control as an oscillating field perturbing a quantum system, with arousal modeled as the strength of the oscillating field, analogous to cortical gain. As illustrated in Figure 3, OFP, when combined with MPMW, successfully reproduces empirical findings about the Yerkes-Dodson law. To the best of our knowledge, this is the first model to comprehensively integrate the Yerkes-Dodson law within a cognitive control framework.

Background

We begin by conceptually reviewing key empirical findings related to the Yerkes-Dodson law, which we aim to explain using the oscillating field perturbation model (OFP).

The Yerkes-Dodson law traditionally posits an inverted U-shaped relationship between arousal and performance: as arousal increases, performance initially improves but eventually declines beyond an optimal point. In the original study by Yerkes and Dodson (1908), arousal was manipulated via caffeine intake in rats, while performance was measured by their success in locating a reward. Later studies with human participants have similarly used caffeine as a proxy for arousal (Teigen, 1994; Anderson, 1994; Neiss, 1988), though other indicators, such as eye movement patterns, have also been employed (DiGirolamo, Patel, & Blaukopf, 2016). Performance has been assessed across a range of domains, including memory recall (Kahneman, 1973), motor coordination (Neiss, 1988), and decision accuracy (Aston-Jones & Cohen, 2005).Since different studies measure arousal and performance in varying ways, we will refer to "arousal" and "performance" in general terms. Despite variability in how both arousal and performance are measured, most studies generally support the inverted U-shaped pattern.

However, the inverted U-shaped characterization of the Yerkes-Dodson Law has been challenged on several fronts. First, some studies report a monotonic relationship between arousal and performance, particularly in tasks with low cognitive demands (Hull, 1943; Zajonc, 1965; Anderson, 1990, 1994). For example, Anderson (1994) found that performance on a simple letter cancellation task improved steadily with increased arousal, while more complex tasks, such as English proficiency assessments, followed the traditional inverted U-shaped pattern. Second, other studies suggest that practice and task familiarity can widen the range of arousal levels that support near-optimal performance (Teigen, 1994; Kahneman, 1973). Figure 3 illustrates how the shape of the Yerkes-Dodson curve varies with task familiarity and difficulty. Finally, a smaller body of research reports anomalous patterns that deviate from both monotonic and inverted U-shaped forms. Some findings suggest that performance may drop off sharply once arousal exceeds a critical threshold, rather than declining gradually as the classic model predicts (Hardy, 1996; Aston-Jones & Cohen, 2005). Others, such as Watters et al. (1997), even report oscillatory patterns of performance as a function of arousal, particularly in tasks influenced by substances like caffeine (Watters, Martin, & Schreter, 1997).

The goal of the OFP model is to offer a unified explana-

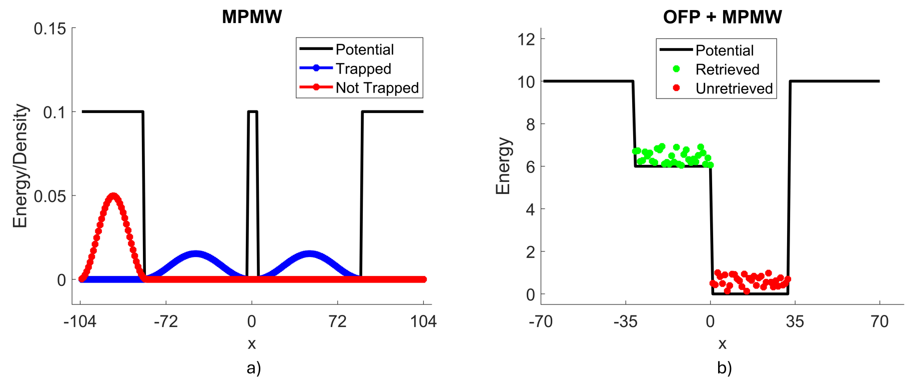

Figure 2: **Illustration of the MPMW and OFP models.** a) **The MPMW model.** In this model, each well represents a choice, with the probability of detecting a particle within a well determining the corresponding choice probability. The plot illustrates the position-space probability distribution of a particle trapped in a symmetric double well potential (blue curve) compared to one that is not trapped (red curve). The y-axis represents both energy and probability density for visualization purposes. b) **Integration of OFP and MPMW.** In the "memory well" potential shown, the deeper well represents long-term memory, and the shallower well represents working memory. An external oscillating field excites particles from the long-term to the working memory well, with the probability of particles in the shallower well indicating the likelihood of successful retrieval.

tion that captures both the classical inverted U-shape of the Yerkes-Dodson Law and the empirical variations in its form as a function of task familiarity and difficulty. To set the stage, we next present a conceptual overview of the Multiple Particle Multiple Well (MPMW) model (Rosendahl et al., 2020; Rosendahl & Cohen, 2022), which serves as the theoretical foundation for the OFP framework. We then highlight key limitations of the MPMW model in accounting for the observed shifts in the shape of the Yerkes-Dodson law.

Multiple Particle Multiple Well

In the MPMW model, decision options are represented as quantum potential wells, with the probability of selecting a specific choice corresponds to the likelihood of detecting particles within these wells. Particles become trapped in these wells and settle at specific energy levels, which determine their detection probabilities.

Formally, the MPMW model represents arousal as the energy eigenvalues E_i of the time-independent Schrödinger equation:

$$H(x)\psi_i(x) = E_i\psi_i(x), \tag{1}$$

where $H(x)$ is the system Hamiltonian (a matrix) of the particles, and $\psi_i(x)$ are the quantum eigenstates (vectors) representing the "energy levels" the particles can occupy.

A simple example of $H(x)$, the symmetric double-well potential, is illustrated in Figure 2a. Conceptually, each well represents a choice, with its width corresponding to the familiarity of that option and its domain generality in semantic space, and its depth represents the strength of current attentional allocation to the choice.

To illustrate the symmetric double-well potential, consider a 2AFC task, in which the participates see a field of randomly moving dots, a fraction of which are moving coherently either left or right, and they must indicate the direction of motion. When properly attending to the task, participants will likely respond with one of the two directions ("left" or "right,") as represented by the blue curve. However, if they do not attend at all to the task, they may produce an entirely incorrect response falling outside both wells, such as "up", illustrated by the red curve. The depths of the wells determine how much attention is allocated to each option, while the widths represent how general the concepts "left" and "right" are.

With the solution to the time-independent Schrödinger equation, we can compute the detection probability that the particle is within the spatial region of a particular well as:

$$P(\text{detect}) = \sum_{i=1}^{N} P(i) \cdot \int_L |\psi_i(x)|^2 \, dx, \tag{2}$$

where $P(i)$ specifies the distribution over the eigenstates ψ_i, L denotes the spatial region of the well, and the integral gives the probability of the particle being within the well at a given eigenstate. $|\psi_i(x)|^2$ is used because quantum probabilities are derived from the squared magnitude of the quantum state. The MPMW model assumes $\sum P(i) = 1$, as it assumes that untrapped particles cannot be detected.

Rosendahl and Cohen (2022) demonstrated that the MPMW model effectively captures key findings in perceptual decision-making tasks. However, when applied to the Yerkes-Dodson law, the model is limited to producing only the catastrophic drop version of the curve (see Figure 1c), where performance declines sharply to zero outside of a specific "working range." This sharp decline arises directly from the constraint that $\sum P(i) = 1$. However, if this constraint is removed and unbounded states are allowed, performance would instead decrease monotonically as a function of energy eigenvalues (arousal), failing to account for any version of the Yerkes-Dodson law. Besides, more importantly, while the well depths and well widths in MPMW can correlate with task difficulty and familiarity, changing them do not alter the shape of the Yerkes-Dodson law. Therefore, MPMW cannot explain various empirical findings on how familiarity and task difficulty influence the shape of the Yerkes Dodson law.

Perturbing MPMW with Oscillating Field

To address the limitations of the MPMW model, we introduce the Oscillating Field Perturbation (OFP) model. In this framework, cognitive control induced by arousal is represented as an oscillating field that excites particles, shifting them from lower-energy eigenstates to higher-energy eigenstates. The model is conceptually inspired by neural oscillatory activity in the LC and adaptive gain theory. However, it is important to emphasize that OFP serves only as a conceptual analogy to these models rather than directly modeling neural mechanisms.

Formally, unlike MPMW which utilizes time-independent Hamiltonians, the OFP model employs a time-dependent Hamiltonian:

$$H_p(t,x) = \begin{cases} H_0(x), & \text{if } t < t_0, \\ H_0(x) + \sum_{n=1}^{N-1} A\cos\big(\omega_n(t-t_0)\big)\cdot x, & \text{if } t \geq t_0. \end{cases}$$

Here, $H_0(x)$ represents the initial Hamiltonian of the system. The term $A\cos(\omega_n(t-t_0))$ describes the oscillating field perturbations, where all oscillating fields share the same amplitude A, but differ in their angular frequencies ω_n. The angular frequencies ω_n are defined as[1]:

$$\omega_n = E_{n+1} - E_n, \quad \omega_n > 0, \tag{3}$$

where E_n and E_{n+1} are the bounded energy eigenvalues of the initial Hamiltonian $H_0(x)$. This choice of angular frequencies satisfies the resonant condition[2] and ensures that the model facilitates transitions exclusively between neighboring states, simplifying computations.

In this work, we employ a specific double-well potential from MPMW as the initial Hamiltonian:

$$H_0(x) = -\frac{\partial^2}{\partial x^2} + \begin{cases} 0, & \text{if } 0 \leq x \leq L_1, \\ V_{\text{work}}, & \text{if } -L_2 \leq x \leq 0, \\ V_{\max}, & \text{otherwise.} \end{cases} \tag{4}$$

Here, $L_1 < L_2$ represent the respective well widths, and $V_{\max} > V_{\text{work}}$ denote the depths of the deeper and shallower wells. Figure 2b provides a visualization of this potential.

It is important to note that while $H_0(x)$ is formulated as a multiple-well potential in the current approach, this is not a necessary requirement in the most general case. We use this specific form of H_0 to illustrate how our framework extends the existing MPMW model. As we will briefly mention in the General Discussion, the OFP model can also be integrated with potentials that are not multiple wells.

Conceptually, the deeper well represents the "long-term memory" well, while the shallower well represents the "working memory" well. Memory retrieval is modeled as the excitation of particles from the deeper long-term memory well to the shallower working memory well induced by the oscillating field. The width of the working memory well relative to the long-term memory well represents task familiarity, analogous to how width is conceptualized in MPMW, where a larger L_2 indicates greater task familiarity. The relative depth, on the other hand, corresponds to task difficulty, with a larger V_{work} representing an easier task. This definition of task difficulty is intuitive, as it is easier for particles to transit from the "long-term memory" well to a deeper "working memory" well. It also aligns with how MPMW conceptualizes depth, as simpler tasks generally require lower cognitive load and thus can achieve higher attentional efficacy. We will refer to this specific double-well potential as the "memory well". It is important to note, however, that the wells in the model do not necessarily correspond precisely to long-term or short-term memory. Their interpretation may vary depending on the specific task. What matters more is the model's ability to reproduce empirical patterns in the specific task.

Since the Hamiltonian is now time-dependent, we must solve the time-dependent Schrödinger equation:

$$\frac{d\psi(t)}{dt} = H_p(t,x)\psi(t)$$

to determine the time-dependent solution $\psi(t)$ used to compute quantum probabilities. Solving this equation becomes easier when the Hamiltonian is expressed in the basis of the bounded energy eigenstates of H_0. In this basis, H_0 is represented as a diagonal matrix, with each energy eigenvalue of H_0, denoted as E_i, appearing on the diagonal. To express H_0 in this basis, one must solve the time-independent Schrödinger equation in Equation 1 and obtain the bounded states, following the same approach as in MPMW.

According to Fujii (2013), when an oscillating field is applied at $t_0 = 0$ (without loss of generality), the perturbed Hamiltonian can be approximated (assuming detuning effects are negligible) in the bounded energy eigenstates basis as:

$$H_p(t) = \begin{bmatrix} E_0 & g\cos(\omega_1 t) & 0 & \ldots & 0 \\ g\cos(\omega_1 t) & E_1 & g\cos(\omega_2 t) & \ldots & 0 \\ 0 & g\cos(\omega_2 t) & E_2 & \ldots & 0 \\ . & . & . & \ldots & . \\ 0 & 0 & 0 & \ldots & E_N \end{bmatrix},$$

where $g > 0$ is the coupling strength constant, directly proportional to the amplitude of the oscillating field A: stronger fields lead to linearly greater coupling. **In OFP, we will use g to represent arousal.** Conceptually, this definition is similar to cortical gain in Adaptive Gain Theory (Aston-Jones & Cohen, 2005), as g is the maximum periodic "energy gain" in the quantum system.

[1]Throughout this paper, we assume that all quantities are unitless, as the specific physical magnitudes of these values are not of interest in the context of our analysis.

[2]In physics, transitions between energy eigenstates occur only when the angular frequency of an oscillating field matches the energy gap between states.

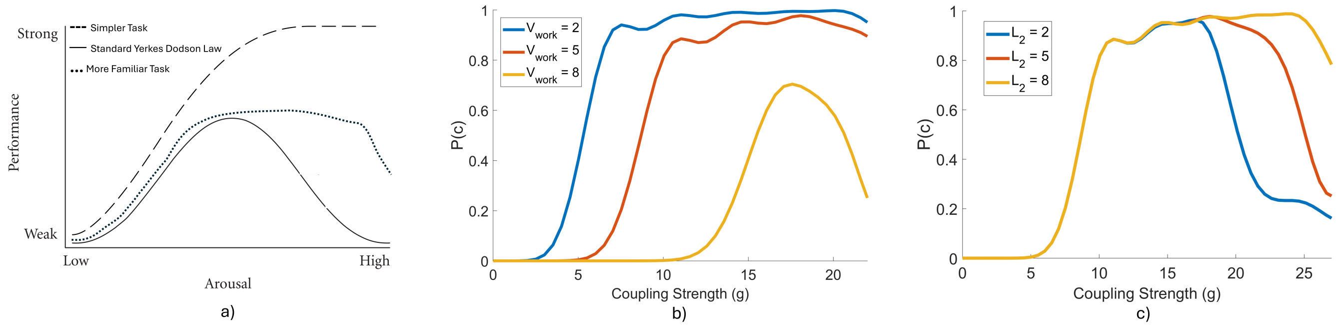

Figure 3: **Predictions of OFP compared with empirical findings.** In (a), we illustrate qualitatively how the shape of the Yerkes-Dodson law is expected to change with variations in task difficulty and familiarity. While the standard Yerkes-Dodson law exhibits an inverted U-shape, performance monotonically increases with arousal for simpler tasks. For more familiar tasks, despite reaching similar performance levels, participants' performance tends to be more resilient to increases in arousal (e.g., they are less likely to become anxious about a task they are more familiar with). In (b) and (c), we present $P(c)$ (performance) as a function of g (arousal) for various values of L_2 (familiarity) and V_{work} (task difficulty) of the OFP model with the memory well potential. A larger L_2 indicates a more familiar task, while a larger V_{work} signifies an easier task. In both (b) and (c), we set $V_{\max} = 10$ and $L_1 = 10$.

A challenge arises from the fact that the time-dependent Schrödinger equation with the above H_p does not have an analytical solution (Shirley, 1965). To address this challenge, we can use the rotating wave approximation (Negele, 2018; Fujii, 2013), and obtain the solution:

$$\psi(t) = e^{-iH_0 t} e^{-iH_R t} \psi(0) \tag{5}$$

$$H_R = \begin{bmatrix} 0 & \frac{g}{2} & 0 & \dots & 0 \\ \frac{g}{2} & 0 & \frac{g}{2} & \dots & 0 \\ 0 & \frac{g}{2} & 0 & \dots & 0 \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}. \tag{6}$$

In the above, $\psi(0)$ represents the initial state. Typically, the initial state $\psi(0)$ is chosen to be the ground state with eigenenergy E_0, as particles naturally settle into the lowest available energy state.

Suppose the process evolves for a fixed unit time $t = 1$. The probability that the particle is at the energy level of the "working memory" well is given by:

$$P(c) = |M_c \psi(t=1)|^2, \tag{7}$$

where M_c is a projection measurement matrix that projects $\psi(t)$ onto bounded states where $E_i > V_{\text{work}}$:

$$M_c(i,j) = \begin{cases} 1, & \text{if } i = j \text{ and } V_{\text{work}} \leq E_i \leq V_{\max}, \\ 0, & \text{otherwise.} \end{cases}$$

Note that since we treat time as unitless (see footnote 1), $t = 1$ can be rescaled to represent any arbitrary time interval.

In OFP, we use $P(c)$ to represent the probability of a successful cognitive process (performance), as the goal is to "retrieve" particles containing certain information from the "long-term memory" well into the "working memory" well. $P(c)$ can then serve as a subjective probability that drives an evidence accumulation model, which we will expand more in the general discussion section.

To summarize, the computation of OFP can be broken down into three steps:

1. Solve the time-independent Schrödinger's equation and construct H_0 in the energy basis as a diagonal matrix of the energy eigenvalues of the bounded states.
2. Define g and $\psi(0)$, then use Equations 5 and 6 to solve for $\psi(t=1)$.
3. Define M_c and apply Equation 7 to $\psi(t=1)$ to compute $P(c)$.

Next, we will demonstrate how the OFP model with the specific "memory well" can replicate major empirical findings of Yerkes Dodson Law.

Replicate Empirical Findings concerning the Yerkes Dodson Law

Since $P(c)$ represents the probability of successful memory retrieval and g corresponds to arousal, verifying the model's alignment with the Yerkes-Dodson law is essentially to examine whether $P(c)$ as a function of g can exhibit empirical patterns shown in Figure 3a.

To conduct this analysis, we fixed $V_{\max} = 10$ and $L_1 = 10$ for the memory wells. We considered six conditions: (1) $L_2 \in [2,5,8]$ with $V_{\text{work}} = 5$ fixed, and (2) $V_{\text{work}} \in [2,5,8]$ with $L_2 = 5$ fixed. We then use matrix numerov method (Pillai, Goglio, & Walker, 2012), to numerically solve for H_0, and examined $P(c)$ for various values of g. The results are presented in Figure 3.

As shown in Figure 3, the model qualitatively replicates all key empirical findings. Specifically, OFP predicts that increasing familiarity (L_2) extends the duration for which $P(c)$ remains at its peak as g increases, while increasing task difficulty reduces peak performance and shifts the curve from a monotonic increase to an inverted U-shape.

It is important to note that although the OFP model can reproduce key empirical findings of the Yerkes-Dodson Law, it does so only when the parameter g falls within a specific range—one in which $P(c)$, as an oscillatory function of g, remains within a single period. Outside this range, the model predicts continued oscillations in $P(c)$. While there is some empirical evidence suggesting oscillatory patterns in performance as a function of arousal (Watters et al., 1997), additional research is needed to robustly confirm this phenomenon. Additionally, it becomes necessary to explain why, empirically, g is typically constrained to a limited range. We offer a tentative explanation for in the General Discussion.

General Discussion

In this work, we propose the oscillating field perturbation (OFP) model, which treats human cognitive control by arousal as an oscillating field perturbing a quantum system, with arousal represented by field strength. The OFP model addresses several key limitations of the previous multiple particle multiple well (MPMW) quantum framework, while providing the first formal unified account of the empirical variations of the Yerkes-Dodson law.

Here, we consider potential applications of, and challenges for the OFP model, that can be explored further in future research.

Integrating into Decision Making Models

In the MPMW model, it is assumed that once the detection probability $P(\text{detect})$ computed, individuals engage in an evidence accumulation process in which quantum particles are sampled as evidence to inform a decision. Formally, $P(\text{detect})$ will be the "mean integration efficiency" or subjective choice probability that drives the dynamics of a quantum sequential sampler (Huang, Busemeyer, Ebelt, & Pothos, 2024, 2023; Rosendahl et al., 2020). Analogously, the OFP model can be incorporated into an evidence accumulation framework by substituting $P(\text{detect})$ with $P(c)$. Integrating the OFP model in this way enables us to translate quantum probabilities into observable choice probabilities and to generate response time predictions.

Beyond MPMW: Quantum Heuristics

OFP can also be extended beyond the MPMW framework. For example, it can be used to determine the control bit of controlled unitary operators when H_0 represents a two-level quantum system with eigenstates corresponding to "Yes" and "No." Controlled unitary operators serve as the quantum probabilistic analogy of the "if statement" in classical computing, and are widely used in cognitive modeling (Huang, Zhang, Busemeyer, & Breithaupt, 2022; Huang, Zhang, Xie, Breithaupt, & Busemeyer, 2024). A concrete application of this idea is in quantum recognition heuristics (Kvam & Pleskac, 2017), where $\psi(t)$ in OFP could function as the control bit of a controlled-NOT gate used to model recognition heuristics.

To very briefly illustrate this application, consider a scenario in which participants choose between object A and object B using recognition heuristics (Goldstein & Gigerenzer, 1999; Gigerenzer & Goldstein, 2011). Suppose the participant has never seen object B, making it unrecognizable. According to recognition heuristics, the decision to choose object A or respond randomly depends on whether object A is recognized.

Formally, the final state $\psi(t)$ in OFP for a two-level H_0 is given by (Fujii, 2013):

$$\psi(t=1) = \begin{bmatrix} \cos(g) \\ \sin(g) \cdot e^{-i(\omega + \frac{\pi}{2})} \end{bmatrix}, \quad P(c) = \sin^2(g).$$

Figure 1b visualizes $P(c)$ as a function of arousal g. When $\psi(t)$ is used as the control bit of the controlled unitary operator, the probability of recognizing object A is given by:

$$P(\text{choose A}) = P(c) + \frac{1}{2}(1 - P(c)) = \sin^2(g) + \frac{\cos^2(g)}{2}.$$

Since $P(\text{choose A})$ is now a function of arousal g, the empirical stochasticity of recognition heuristics can be attributed to variations in arousal levels. Specifically, this stochasticity arises from the fact that individuals may not always be optimally aroused to retrieve the relevant information needed to recognize object A. A more detailed exploration of this integration will be addressed in future work.

Oscillations in Performance

In our earlier analysis, we noted that the OFP model can replicate the Yerkes-Dodson law only under the assumption that the process terminates within a single period of $P(c)$ as a function of gain g. This raises two important questions: first, why does gain not continue increasing indefinitely, resulting in performance oscillations; and second, is there empirical evidence that performance actually oscillates with increasing arousal?

A tentative answer to the first question may lie in how arousal—or gain—naturally adapts toward optimal levels. According to Adaptive Gain Theory, arousal is regulated by dynamic adjustments in locus coeruleus (LC) activity to optimize the balance between environmental uncertainty and task demands (Aston-Jones & Cohen, 2005). However, the exact mechanisms that constrain the range of arousal—and prevent it from traversing multiple oscillatory cycles—remain unresolved. As for the second question, to our knowledge, the only study reporting oscillatory performance patterns as a function of arousal is that of Watters et al. (1997), who observed such effects in an English proficiency task under varying levels of caffeine. We encourage further research to explore this more.

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