Problem-solving Strategies in Frictional Force Problems: Evidence from Think-aloud Protocols

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Abstract

Prior work has shown that novices' intuitive conception of frictional force is often different from that of formal physics theory. These discrepancies can lead to challenges in physics education. The current paper focuses on a common misconception: "frictional force always resists motion." Though frictional force is linked to relative motion (motion of one object compared to another), it is not always determined by absolute motion. We collected think-aloud protocols from participants, who had varying levels of prior knowledge, on physics problems about motion, relative motion, and frictional force. We analyzed how participants selected properties, made comparisons, and drew inferences. We found that regardless of prior knowledge, participants were able to extract relevant information from the descriptions. When collapsed across the three problem types, frequencies of comparisons did not differ. However, participants with high prior knowledge were more likely to compare objects when making inferences about force and motion. In contrast, participants with low prior knowledge were more likely to rely on insufficient information like motion of a single object. Participants with high prior knowledge were also more likely to leverage experience with other problems, by comparing across processes or constructing hypothetical scenarios.

Keywords: naïve (intuitive) physics; frictional force; relative motion; think-aloud protocol

Introduction

Many studies show that people's intuitions about physical forces diverge from force concepts in formal theory (e.g., Bliss & Ogborn, 1994; McCloskey, 1983). For example, some college students believe that an internal force is necessary to sustain the motion of objects (Horiguchi et al., 2023; Viennot, 1979), similar to the medieval impetus theory (Halloun & Hestenes, 1985; Hestenes et al., 1992). Such intuitive conceptions can create learning barriers in physics classrooms, persisting even after years of education (Bani-Salameh, 2017).

The current study examines intuitive conceptions of frictional force, which is the force resisting the *relative motion* (the motion of one object *compared to* another) of solid surfaces sliding against each other[1]. Frictional force is one of the topics in introductory physics courses that students find especially challenging (Chia, 1996; Sharma & Sharma, 2007). Recently, a literature review highlighted four types of misconceptions about frictional force: definition and existence, direction, type and magnitude, and effects (Kızılcık et al., 2021). The most common misconception about frictional force is "frictional force always resists (actual motion)." This relation is true for simple cases when there is only one moving object. But it can lead to errors when there are multiple objects and when relative motion does not align with actual motion. Consider this example: when a person is walking on the sidewalk, frictional force acts in the backward direction. But on airport moving walkways, the direction of frictional force can be forward, if the walkway moves faster than the person.

Though this misconception has been reported frequently, there is relatively little work on why students have this misconception. Besson and colleagues assessed causal explanations of friction and other physical phenomenon, and suggested novices have a tendency to delocalize forces, skipping intermediate objects (Besson, 2010; Besson et al., 2007). Other researchers argue that novices use a reasoning pattern called "restrictive rule-based reasoning," which is applying heuristics overconfidently (Cheong et al., 2019). More recently, a study has shown that students with higher levels of knowledge integration perform better when reasoning about friction (Wang et al., 2024).

The purpose of the current study is to analyze the reasoning steps of participants with varying levels of prior knowledge when they solve friction problems. The reasoning steps of physics experts follow a typical pattern (which we summarize based on interviews in a pilot study): First, relevant properties are extracted from the problem description, such as whether there is an external force acting on the objects. Then, they use these properties to reconstruct the physical process. To reason about frictional force, they compare the motion between objects (i.e., relative motion), which leads to their inference of frictional force (the direction of force is opposite relative motion).

There are several ways in which the reasoning behavior of novices can diverge from that of experts. It is possible that novices are unable to distinguish between relevant and irrelevant information in the problem description, which leads to subsequent challenges in reasoning. Moreover, it is

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[1] Here we only consider kinetic friction between solids.

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unclear whether novices make comparisons between objects when reasoning about friction. One possibility is that novices do not make comparisons between objects, but instead they rely on superficial features like speed and external forces. Another possibility is that novices understand the relation between frictional force and relative motion, but encounter difficulties when making comparisons. Such challenges may arise from problems in using vector representations—for example, when performing vector additions and subtractions (Aguirre, 1988; Heckler & Scaife, 2015). Vector representations may be more difficult to use in physics contexts than in math contexts (Shaffer & McDermott, 2005; Van Deventer et al., 2007).

We devised the present experiment to probe the difficulties people have in thinking about friction. Our earlier pilot studies found differences between physicists' methods of solving friction problems (just outlined) and those of novice participants on traditional textbook-type problems. Here we use think-aloud protocols to get a deeper understanding of the source of these discrepancies, following the success of think-alouds in developing models in a variety of reasoning tasks (Ericsson & Simon, 1980; Padilla & Leighton, 2017). Think-aloud protocols have also been used in physics education to assess students' ability to construct and update scientific models (Ríos et al., 2019; Zwickl et al., 2015).

Our guiding research questions are: (a) Are novices able to extract relevant properties from the problem description? (b) What (if any) comparisons do novices make when reasoning about frictional force? (c) What are the characteristics of inferences made by novices?

Method

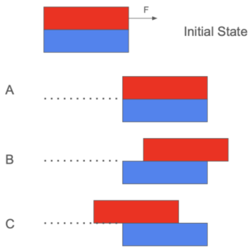
Materials

Participants received practice in thinking aloud on two questions taken from previous work: digit addition and the Tower of Hanoi (Ericsson & Simon, 1980; Kotovsky et al., 1985).

During the think-aloud experiment, participants worked on 14 sets of questions. Each set of questions included one question on motion prediction, two questions on relative motion, and two questions on frictional force. The motion prediction questions described a physical process involving kinetic frictional force, and participants predicted what will happen next by choosing from three possible "snapshots." The relative motion and frictional force questions asked about the direction of motion and force. An example of a problem set is shown below:

[Description] A red block and a blue block are stacked on the ground, and the surface between the two blocks is rough. At t=t0, a horizontal force F is exerted on the red block and causes both blocks to move horizontally, while the two blocks slide against each other.

[Motion prediction] When t=t1, which of the following options best describes the location of the two blocks?



[Relative Motion] When t=t1, what is the moving direction of the blue (red) block relative to the red (blue) block?

[Frictional Force] When t=t1, what is the direction of frictional force exerted on the red (blue) block by the blue (red) block?

Procedure

First, participants read instructions that explained how to think aloud. They also read about the definition of relative motion, illustrated with two everyday examples. Participants then practiced thinking-aloud on the two general domain questions. Then, they read sample questions (which only described one moving object) on motion prediction, relative motion judgment, and frictional force judgment to familiarize them with the tasks in the experiment. During the experiment, they completed 14 sets of questions while thinking aloud. The order of the problem sets was randomized. The experimenter would ask for clarification if a response was incomplete or unclear, but did not give feedback on the accuracy of any question. After the think-aloud session, participants completed a survey on their prior physics knowledge and reasoning strategy. Finally, the experimenter conducted a short (3-10 min) debriefing interview based on the responses.

The audio files from the session were automatically transcribed on Microsoft Word and manually checked to avoid errors. Following Ericsson (2017), the responses were segmented into utterances and then categorized based on the coding scheme described below.

Participants

Sixteen participants were recruited for this experiment. Of these participants, 6 had taken at least one college level physics course, 9 had taken at least one high school level physics course (but no college course), and 1 had never taken a physics course. Due to the relatively small sample size, we will combine the participant with no prior knowledge and the high school level group. This is reasonable since the performance of participants with these knowledge levels had been similar in previous experiments.

Coding Scheme

We were mainly interested in how participants selected properties, made comparisons, and generated inferences during the experiment. Participants may extract properties that may not be relevant for solving the problem. If novices are unable to filter the information, this may lead to difficulties in reasoning. Making comparisons is an important strategy for analyzing given information, and especially relevant for reasoning about relative motion and frictional force. This is because relative motion, by definition, is the motion of one object *compared to* another; and frictional force is based on relative motion. Finally, inferences are also of interest, because they suggest the rules participants are using to reason about relative motion and frictional force. By coding the information just mentioned, we can compare the strategies of more knowledgeable participants to less knowledgeable ones (see Table 1 for examples).

Selected properties After initially reading the description of the question, participants would selectively refer to some of the given properties. These properties included spatial relations between the objects, the initial speed of objects, roughness of surface, and forces applied to objects. Initial speed and force are most relevant in reasoning about frictional forces, because these properties are closely linked to the motion of the objects. Roughness and spatial relations are not directly linked with relative motion or frictional force. The fact that objects are in contact on rough surfaces is only a prerequisite to allow frictional forces to exist. If the participants refer to relevant properties such as initial speed and force more than irrelevant properties, this implies that they are good at extracting information from the descriptions.

Comparison When coding comparisons, we only considered spontaneous comparisons and excluded direct responses to the relative motion questions.

We can differentiate comparison of one object over time, comparison between two objects, and comparison between two processes. Within each of these categories, we can also differentiate numerical and directional comparisons. Note that these types of comparisons are not mutually exclusive. For example, an utterance like "the red block is moving to the left, while the blue block is moving to the right" would be coded as both comparison between objects and directional comparison.

Comparison across time is the most basic type of comparison, in which the participant looks at how the location (or other attributes) of one object changes. Comparison across objects occurs when the attribute of one object is compared to that of another object. Deriving relative motion, by definition, requires the participant to compare the velocity of one object to another. Comparison across processes involves noticing the similarities and differences between different items.

Numerical comparison involves comparing the *magnitude* of one attribute to another, while directional comparison involves comparing the *direction* of one attribute to another. In determining relative motion, if two objects are moving in different directions, then a directional comparison is sufficient; if they are moving in the same direction, then it is necessary to look at the difference in the magnitudes of velocities to determine relative motion.

Inference Only explicit inferences that included both identifiable premises and conclusions were coded in this experiment to avoid ambiguities. Words like "because," "then," "since," and "so" were used as markers for coding inferences. We predicted that participants with high prior knowledge would be more likely to make inferences based on comparison between objects and physical laws.

Alternative strategies Utterances describing alternative strategies like intentions of objects and hypothetical scenarios were also coded post-facto. Some participants stated the intention of objects to explain why there would be frictional forces. These utterances are often marked by phrases like "wants to" or "tries to." Other participants thought about what would happen in hypothetical scenarios and used conclusions from them as a steppingstone to reason about the current problem. For example, when reasoning about the motion of a block with initial speed of 1m/s when put on a conveyor belt, one participant first discussed what would happen if the block had no initial speed. The discussion of hypothetical scenarios can be viewed as a similar strategy to comparison across items. Instead of comparing the current problem with another problem given in the experiment, participants are actively manipulating one or more factors and constructing their own problems.

Table 1. Coding Categories and Examples

Coding Categories		Examples
Selected Properties	Initial speed	The block has an *initial speed* of 5m/s.
	Spatial relation	The red block *is stacked on* the blue block.
	Roughness	The conveyor belt is *rough*.
	External force	There is a *force* pulling the blue block to the right.
Comparison	Comparison across time	Because the initial speed of the block is to the right, it would move to the right *from t0 to t1*.
	Comparison across objects	The blue block has an initial speed of 5m/s, while the red block has an initial speed of 0m/s.
	Comparison across processes	This is *similar to the previous question*, but this time the block has a larger initial speed.
	Numerical comparison	The blue block is moving *faster* than the red block.
	Directional comparison	The blue block would move *further to the right* than the red block.
Inference	Inference of motion	*Because* the force is pulling the block, the block would move to the right.

	Inference of frictional force	The blue block is moving to the right, and frictional force is the only force pulling it, *so* frictional force is to the right.
Alternative strategies	Intention of objects	The red block *wants* the blue block to move right.
	Hypothetical scenario	*If* the surface is smooth and there is no friction, then…

Results

Coding Reliability A second coder (who was not informed of hypotheses) coded transcripts of five participants (2 from the high prior knowledge group) according to a codebook. The average number of coding units in each transcript was 101.8. Interrater reliability was assessed via Cohen's Kappa (Warrens, 2015). We achieved an average Kappa of 0.89 across categories, showing excellent interrater agreement (Fleiss et al., 2013).

Selected Properties This part of the coding is exploratory and there are no predictions for whether we will see differences in the selected properties. Table 2 summarizes the frequency and proportion of the four selected properties. Across these four types of properties, participants referred to initial speed most often, followed by force in both groups of participants. Spatial relations and roughness appeared less frequently. This shows that, in general, participants were able to distinguish the relevance of information in the descriptions. Compared to participants with lower levels of prior knowledge, participants with higher levels of prior knowledge were less likely to refer to initial speed in their reasoning process. This does not imply that participants with higher knowledge were overlooking information about initial speed, because this information can be used indirectly, for example by comparing the speeds of two objects.

Table 2. Descriptive statistics of selected properties

Prior Knowledge	M (SD) averaged across participants		Proportion (%)	
	High	Low	High	Low
Initial speed	2.83 (2.6)	5.40 (3.8)	44	58
Spatial relation	1.00 (2.0)	0.6 (1.1)	15	6
Roughness	0.67 (0.8)	0.70 (1.1)	10	8
External force	2.00 (0.9)	2.60 (1.5)	31	28

Comparisons We hypothesized that participants with more prior knowledge would make a larger number of comparisons across objects and across processes. We also expected to see that participants with less prior knowledge would find directional comparisons especially challenging.

Frequency and proportion of the comparison types are summarized in Table 3. The two groups made approximately the same number of comparisons ($\chi^2(4) = 3.81, p = 0.44$). Comparison of the same object across time rarely occurred, presumably because this type of comparison is simple and quickly assessable in most cases. As expected, comparison across objects was the most frequent type of comparison. Participants with high prior knowledge made qualitatively more comparisons across processes, out of all comparisons (14% vs. 9%). We observed that participants with high prior knowledge were able to notice similarity between processes and use this as a mental shortcut. For example, one participant noticed the similarity between acceleration and deceleration and said, "This is basically the opposite of the last question. So box relative to the car is left; (car) relative to the box is right…"

Table 3. Descriptive statistics of comparisons

Prior Knowledge	M (SD)		Proportion (%)	
	High	Low	High	Low
Across time	0 (0.0)	0.50 (1.1)	0	4
Across object	8.0 (4.2)	10.4 (4.0)	86	87
Across processes	1.3 (1.1)	1.2 (1.5)	14	9
Numerical	4.17 (2.8)	4.80 (2.7)	48	47
Directional	4.16 (2.7)	5.70 (2.1)	52	53

We also observed that participants in the low prior knowledge group often made errors in directional comparisons but were accurate in numerical comparisons. Consider the following excerpt: "It (the block) has speed, separate to the conveyor belt, so it moves faster than the belt. So *it's moving to the right compared to the belt*, and *the belt is moving to the right of the block*, just at slower speed…" The participant started with a numerical comparison then ended up with a contradiction (utterances shown in italic) when reasoning about the direction of relative motion.

Inference For the high prior knowledge group, there was a higher frequency of force inferences (*M*=9.67 per participant, *SD* = 6.5), compared to motion inferences (*M*=3.5, *SD*=2.1). A similar trend was observed for the low prior knowledge group (force: *M*= 12.6, *SD* = 10.8; motion: *M*=1.9, *SD* = 2.0).

We further assessed the validity and accuracy of inferences. Valid inferences are inferences that include *sufficient* information in the premises to arrive at the correct conclusion. Correct inferences are inferences that arrive at the correct conclusion regardless of the information in the premises. For example, if we consider the sample question shown in Materials, an inference like "the frictional force is to the left, because the blue block moves right relative to the red block" would be valid but incorrect; "the frictional force is to the right, because the red block is moving to the right" would be invalid but correct. We hypothesized that participants with high prior knowledge would achieve higher accuracy and validity in inferences across the board. The results are summarized in Table 4.

We observed that participants with high prior knowledge achieved high accuracy in inferences about force and motion. Participants with low prior knowledge are relatively accurate in inferences about forces but perform poorly in inferences about motion. Group differences were analyzed with chi-squared tests. Results show that participants with high prior knowledge achieved higher validity ($\chi^2(3) = 4.42, p = 0.012$) and higher accuracy in inferences about force ($\chi^2(3) = 7.58, p = 0.005$). For inference about motion, no statistical difference was found due to the small number of inferences ($\chi^2(3) = 0.61, n.s.; \chi^2(3) = 0.19, n.s.$).

Furthermore, our results show that it's challenging for participants to make explicitly valid inferences during a think-aloud experiment, whether they have a high or low level of prior knowledge. This gap between the accurate and valid inferences can be explained, in part, by the fact that some information was implicitly used in the reasoning process but not stated, especially when the information is obvious. Constraints on cognitive resources may make it difficult to verbalize all the premises of an inference.

Table 4. Proportion of valid and accurate inferences

Prior Knowledge	Accuracy (%)		Validity (%)	
	High	Low	High	Low
Inferences of Motion	70	50	55	39
Inferences of Force	98	84	50	37

In addition, we categorized the types of information in the premises. Possible categories of information included initial speed, external force, motion of target object, motion of the other object, comparison between objects, frictional force of the other object, intention. Some of the categories rarely appeared and were dropped from further analysis.

For inferences about motion, we found that participants with low prior knowledge were more likely to infer motion based on initial speed, whereas participants with high prior knowledge were more likely to infer motion based on forces and on comparisons between objects. Note that the total number of motion inferences was relatively small, and the content of the inferences partly depend on the context of the problems; so we caution against drawing general conclusions from these trends. One erroneous inference that repeatedly appeared in the low prior knowledge group was "Because the block is not moving (initially), it will just go along with the conveyor belt (at the same speed)."

Table 5. Type of premise in inference about motion

Prior Knowledge	Proportion (%)	
	High	Low
Initial speed	25	46
External force	35	11
Comparison between objects	20	14

For inferences about force, the distribution of premises in the two participant groups was different $\chi^2(4) = 27.49, p < 0.001$). For both groups, the most common premise types were descriptions of the target object's motion (the motion of the object that the question asks about) or the other object's motion. However, participants with high prior knowledge were more likely to use premises that compared the two objects (e.g., comparing the speed or location of the objects) and to use premises that described the frictional force of the other object ($\chi^2(1) = 5.7, p = 0.02$). We speculate that these premises were based on the formal definition of frictional forces and application of Newton's law. It's surprising that participants with high prior knowledge did not mention definitions or laws from physics theory more often than motion information in the premises. These results are summarized in Table 6.

Table 6. Type of premise in inference about force

Prior Knowledge	Proportion (%)	
	High	Low
Motion of target object	33	37
Motion of other object	17	23
Comparison between objects	17	11
Frictional force of other object	17	5

Alternative Strategies We coded for two alternative strategies: describing the intention of objects and hypothetical scenarios. Participants from the low prior knowledge group seemed more likely to describe the intention of objects. These utterances: a) informally described the interaction between two objects, or b) described the innate tendency of an object's motion. For example, "the car wants to hold the box in place" describes the interaction between objects, and "the block wants to stay in place" describes the innate tendency of the object. We found that in the high prior knowledge group, all such utterances referred to interactions between objects; however, in the low prior knowledge group, 75% of utterances described innate tendencies, especially initially static objects wanting to remain still. These utterances are likely examples of anthropomorphism (Caporael, 1986; Epley et al., 2007).

The discussion of hypothetical scenarios was rare in both groups. The most common topic of these scenarios was the roughness of the surface, where participants reasoned about what would happen if the surface was very rough or very smooth. Some participants also discussed how the physical process might change when varying the initial speeds. For example, some participants treated "object with initial speed of 0" as a base case and used this base case to reason about other scenarios. Discussing hypothetical scenarios was helpful overall, because the participants were able to start from rather simple examples and make inference from there. But there were also cases where they started with a hypothesis but failed to keep track of it. Here is an unsuccessful example: "I'm wondering *if the friction will cause the red block to actually move to the right a bit* [hypothesis]. And if that's the case, then the friction would be to the left, but if the red block, the red would not stay in place. The friction would drag the red block. So I guess the *frictional force*, if the red block is moving to the right, *would be to the left then* [conflicting conclusion]." The participant starts with the hypothesis "friction on the red block is to the right." They then infer that "the red block would not stay in place," and subsequently conclude that "friction would be to the left" to oppose motion. This conclusion contradicts the hypothesis, but the participant was unable to detect this conflict.

General Discussion

In the current study, we collected think-aloud protocols on questions about motion, relative motion, and frictional force. Overall, our analysis highlights the commonalities and differences of reasoning strategies between participants with varying levels of prior knowledge. Frequencies of mention of selected properties were similar across groups, with relevant properties like initial speed referred to more frequently than irrelevant properties (Table 2). This shows that participants in both groups were able to identify relevant information.

We also analyzed the frequencies of comparisons in both groups (Table 3). Comparison across objects was the most common type of comparison in both groups, followed by comparison across processes and comparison across time. To our surprise, we found that both groups generated the same number of comparisons across objects. Differences emerged, however, when we analyzed the comparisons' use. Participants with low prior knowledge were less likely to mention comparisons between objects when making inferences about motion and force (Table 5, 6). This suggests that participants with low prior knowledge were less likely to understand the link between frictional force and relative motion, even though each problem set contained questions that explicitly asked them to reason about relative motion. They were also less accurate in directional comparisons compared to numerical comparisons, consistent with prior work showing difficulties in using vector representations in physics (Shaffer & McDermott, 2005; Van Deventer et al., 2007). In addition, participants with high prior knowledge were more likely to make comparisons across physical processes. This is an example of reasoning via case comparisons, and it suggests that these participants have schemas they applied to similar processes, instead of reasoning about friction on a case-by-case basis (Alfieri et al., 2013).

We further examined participants' inferences about force and motion. We observed that participants with low prior knowledge were less likely to produce accurate or valid inferences. Regardless of prior knowledge level, it was more challenging to produce valid inferences than accurate inferences, perhaps because of constraints on cognitive resources during think-aloud experiments (Ericsson & Simon, 1980). When reasoning about frictional force, participants with high prior knowledge were more likely to use premises based on definitions or physics laws. However, motion information of one object accounted for about half of the premises, even though this information is not sufficient to infer the direction of frictional force.

Finally, participants from both groups referred to the intention of the objects with phrases like "want" and "try to," though the connotation was different. For participants with high prior knowledge, this was mainly used as an informal way of saying one object is exerting a force on the other. For participants with low prior knowledge, such utterances mostly referred to the innate tendency (similar to inertia) of objects. Such descriptions of object intentions is consistent with prior work connecting commonsense beliefs of force and motion via "effort," where effort is required to initiate and sustain motion (Bliss & Ogborn, 1994). Moreover, participants with high prior knowledge were more likely to reason about hypothetical scenarios, often varying one factor continuously while holding other factors constant. This strategy can be regarded as a more complex version of making comparisons across processes (Evans et al., 2003).

Conclusion

Frictional force is one of the challenging topics in introductory physics courses (Sharma & Sharma, 2007). In the current study, we examined how participants with varying levels of prior knowledge reasoned about problems related to frictional force while thinking aloud. Motivated by interviews from a pilot study, we focused on how participants with low or high prior knowledge selected properties, made comparisons, and drew inferences. We found that participants in both groups were able to identify relevant information. When collapsing across problem type, the frequency of comparisons also did not differ. However, participants with high prior knowledge were more likely to mention comparisons between objects when making inferences about motion and force. We also observed that participants with high prior knowledge were more accurate in making directional comparisons. Moreover, they were more likely to leverage experiences with other problems by making comparisons across processes or constructing hypothetical scenarios. Together, these results highlight general similarities between reasoning strategies and suggest that differences in performance may be due to difficulties in linking frictional force with relative motion, and making directional comparisons.

References

Aguirre, J. M. (1988). Student Preconceptions about Vector Kinematics. *Physics Teacher*, *26*(4), 212–216.

Alfieri, L., Nokes-Malach, T. J., & Schunn, C. D. (2013). Learning Through Case Comparisons: A Meta-Analytic Review. *Educational Psychologist*, *48*(2), 87–113. https://doi.org/10.1080/00461520.2013.775712

Bani-Salameh, H. N. (2017). How persistent are the misconceptions about force and motion held by college students? *Physics Education*, *52*(1), 014003. https://doi.org/10.1088/1361-6552/52/1/014003

Besson, U. (2010). Calculating and Understanding: Formal Models and Causal Explanations in Science, Common Reasoning and Physics Teaching. *Science & Education*, *19*(3), 225–257. https://doi.org/10.1007/s11191-009-9203-9

Besson, U., Borghi, L., De Ambrosis, A., & Mascheretti, P. (2007). How to teach friction: Experiments and models. *American Journal of Physics*, *75*(12), 1106–1113. https://doi.org/10.1119/1.2779881

Bliss, J., & Ogborn, J. (1994). Force and motion from the beginning. *Learning and Instruction*, *4*(1), 7–25. https://doi.org/10.1016/0959-4752(94)90016-7

Caporael, L. R. (1986). Anthropomorphism and mechanomorphism: Two faces of the human machine. *Computers in Human Behavior*, *2*(3), 215–234. https://doi.org/10.1016/0747-5632(86)90004-X

Cheong, Y. W., Ha, S., Byun, T., & Lee, G. (2019). Two patterns of student reasoning in problem solving concerning frictional force. *Physics Education*, *54*(2), 025009. https://doi.org/10.1088/1361-6552/aaf6f9

Chia, Ch. T. (1996). Common Misconceptions in Frictional Force among University Physics Students. *Teaching and Learning*, *16*(2), 107–116.

Epley, N., Waytz, A., & Cacioppo, J. T. (2007). On seeing human: A three-factor theory of anthropomorphism. *Psychological Review*, *114*(4), 864–886. https://doi.org/10.1037/0033-295X.114.4.864

Ericsson, K. A. (2017). Protocol Analysis. In W. Bechtel & G. Graham (Eds.), *A Companion to Cognitive Science* (1st ed., pp. 425–432). Wiley. https://doi.org/10.1002/9781405164535.ch33

Ericsson, K. A., & Simon, H. A. (1980). Verbal reports as data. *Psychological Review*, *87*(3), 215–251. https://doi.org/10.1037/0033-295X.87.3.215

Evans, J. St. B. T., Over, D. E., & Handley, S. J. (2003). A Theory of Hypothetical Thinking. In D. Hardman & L. Macchi (Eds.), *Thinking: Psychological Perspectives on Reasoning, Judgment and Decision Making* (1st ed., pp. 1–21). Wiley. https://doi.org/10.1002/047001332X.ch1

Fleiss, J. L., Levin, B., & Paik, M. C. (2013). *Statistical Methods for Rates and Proportions* (3rd ed). Wiley.

Halloun, I. A., & Hestenes, D. (1985). Common sense concepts about motion. *American Journal of Physics*, *53*(11), 1056–1065. https://doi.org/10.1119/1.14031

Heckler, A. F., & Scaife, T. M. (2015). Adding and subtracting vectors: The problem with the arrow representation. *Physical Review Special Topics - Physics Education Research*, *11*(1), 010101. https://doi.org/10.1103/PhysRevSTPER.11.010101

Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *The Physics Teacher*, *30*(3), 141–158. https://doi.org/10.1119/1.2343497

Horiguchi, T., Hirashima, T., & Hayashi, Y. (2023). Error-based simulation as a thought experimental method for changing “motion implies a force” misconception: An evaluation. *Journal of Computer Assisted Learning*, *39*(4), 1290–1302. https://doi.org/10.1111/jcal.12800

Kızılcık, H. Ş., Aygün, M., Şahin, E., Önder-Çelikkanlı, N., Türk, O., Taşkın, T., & Güneş, B. (2021). Possible misconceptions about solid friction. *Physical Review Physics Education Research*, *17*(2), 023107. https://doi.org/10.1103/PhysRevPhysEducRes.17.023107

Kotovsky, K., Hayes, J. R., & Simon, H. A. (1985). Why are some problems hard? Evidence from Tower of Hanoi. *Cognitive Psychology*, *17*(2), 248–294. https://doi.org/10.1016/0010-0285(85)90009-X

McCloskey, M. (1983). Intuitive Physics. *Scientific American*, *248*(4), 122–130. https://doi.org/10.1038/scientificamerican0483-122

Padilla, J.-L., & Leighton, J. P. (2017). Cognitive Interviewing and Think Aloud Methods. In B. D. Zumbo & A. M. Hubley (Eds.), *Understanding and Investigating Response Processes in Validation Research* (Vol. 69, pp. 211–228). Springer International Publishing. https://doi.org/10.1007/978-3-319-56129-5_12

Ríos, L., Pollard, B., Dounas-Frazer, D. R., & Lewandowski, H. J. (2019). Using think-aloud interviews to characterize model-based reasoning in electronics for a laboratory course assessment. *Physical Review Physics Education Research*, *15*(1), 010140. https://doi.org/10.1103/PhysRevPhysEducRes.15.010140

Shaffer, P. S., & McDermott, L. C. (2005). A research-based approach to improving student understanding of the vector nature of kinematical concepts. *American Journal of Physics*, *73*(10), 921–931. https://doi.org/10.1119/1.2000976

Sharma, S. V., & Sharma, K. C. (2007). Concepts of force and frictional force: The influence of preconceptions on learning across different levels. *Physics Education*, *42*(5), 516–521. https://doi.org/10.1088/0031-9120/42/5/012

Van Deventer, J., Wittmann, M. C., Hsu, L., Henderson, C., & McCullough, L. (2007). Comparing Student Use of Mathematical and Physical Vector Representations. *AIP Conference Proceedings*, 208–211. https://doi.org/10.1063/1.2820935

Viennot, L. (1979). Spontaneous Reasoning in Elementary Dynamics. *European Journal of Science Education*, *1*(2), 205–221. https://doi.org/10.1080/0140528790010209

Wang, B., Han, W., Zhang, Y., Wang, Q., Li, D., Tang, Z., & Kong, Q. (2024). Assessment of student knowledge integration in learning friction force. *Journal of Baltic Science Education*, *23*(4), 767–785. https://doi.org/10.33225/jbse/24.23.767

Warrens, M. J. (2015). Five Ways to Look at Cohen's Kappa. *Journal of Psychology & Psychotherapy*, *05*(04). https://doi.org/10.4172/2161-0487.1000197

Zwickl, B. M., Hu, D., Finkelstein, N. D., & Lewandowski, H. J. (2015). Making Models of Measurement Tools: Examples from Think-Aloud Student Interviews. *2014 Physics Education Research Conference Proceedings*, 291–294. https://doi.org/10.1119/perc.2014.pr.069