

# The Impact of Immediate and Elaborative Feedback on Second Grade Students' Equation Solving and Understanding of the Equal Sign

**Anna N. Bartel (abartel@wested.org)**

WestEd  
San Francisco, CA, United States

**Jacklyn Power (jpowers@wested.org)**

WestEd  
San Francisco, CA, United States

**Amy L. Miyahara (amiyahar@nd.edu)**

University of Notre Dame  
Notre Dame, IN, United States

**Yvonne Kao (ykao@wested.org)**

WestEd  
San Francisco, CA, United States

**Jodi Davenport (jdavenp@wested.org)**

WestEd  
San Francisco, CA, United States

**Nicole M. McNeil (nmcneil@nd.edu)**

University of Notre Dame  
Notre Dame, IN, United States

## Abstract

Understanding mathematical equivalence is critical for students' success in algebra. Despite its importance, many students misinterpret the equal sign due to early exposure to operational patterns in arithmetic, leading to entrenched misconceptions. This study investigates how feedback can correct these misconceptions through the implementation of an online version of the Improving Children's Understanding of Equivalence materials. The study used A/B testing to evaluate accumulating and diminishing feedback on second graders' understanding of mathematical equivalence. Students made significant improvements in equation solving and conceptual understanding across both feedback types, underscoring the value of immediate, elaborative feedback. However, no significant difference was observed between feedback conditions. The present study informs the development of effective instructional interventions for early mathematics education that can be delivered at scale. By refining automated feedback and addressing student-specific learning needs, educators and researchers can strengthen foundational mathematical understanding and better prepare students for future algebraic success.

**Keywords:** mathematical equivalence; immediate feedback; elaborative feedback; conceptual knowledge; problem solving; classroom studies

## Introduction

Success in algebra is a key predictor of access to future educational and employment opportunities (Ma, 2001).

However, 75% of U.S. students fail to reach basic math proficiency by the end of the 8th grade (NAEP, 2022). What foundational knowledge equips students for later success in algebra? Understanding mathematical equivalence, the concept that the quantities on either side of an equal sign are interchangeable, is crucial for predicting later mathematics achievement (Hornburg, Devlin, McNeil, 2022; Matthews & Fuchs, 2020; McNeil et al., 2019) and is essential for developing early algebraic thinking (Alibali et al., 2007).

In early elementary education, typical arithmetic equations, such as  $2 + 3 = \underline{\quad}$ , present operations on the left and the answer on the right of the equal sign (McNeil et al., 2006; Powell, 2012). Overuse of this equation format leads students to interpret the equal sign as a cue for action rather than as a relational symbol between quantities. Children generalize erroneous "operational patterns," such as interpreting the equal sign as "find the answer," to future math contexts (Jacobs et al., 2007). This misunderstanding becomes evident in equations with operations on both sides of the equal sign, such as  $5 + 4 = \underline{\quad} + 3$ , where students might add only the numbers before the equal sign or all numbers in the equation (Alibali & Goldin-Meadow, 1993; McNeil, 2007; McNeil & Alibali, 2004; Perry, Church, & Goldin-Meadow, 1988; Rittle-Johnson & Alibali, 1999).

Students may continue to activate and rely on operational patterns during practice and fail to get timely feedback about their mistakes. Students who erroneously apply operational patterns may feel confident and be unlikely to ask for help or

pay attention when teachers review problems (Nelson & Fyfe, 2019). As students continue to practice with errors, the operational patterns become more entrenched over time. This is an issue because early reliance on operational patterns will eventually require additional support for understanding mathematical equivalence (McNeil et al., 2019).

### Accumulating and Diminishing Feedback

Providing feedback during problem solving may help students shift from the incorrect strategies described above to correct ones (Fyfe et al. 2012). For younger students, providing elaborative feedback that explains a mistake is more effective than accuracy feedback that simply tells students an error occurred (Hornburg, 2017). As students may be reinforcing incorrect understandings each time they make an error, feedback must be timely and understandable. Computer-based feedback has the potential to meet both objectives.

Studies of computer-based feedback often target older students (i.e., middle school or above). However, elementary school teachers and young students may find computer-based feedback to be just as effective as teachers and students in older grades (Moyer-Packenham et al., 2019; Van der Kleij et al., 2015). Computer-based feedback may be particularly useful in a classroom setting because it is nearly impossible for teachers to provide *immediate* feedback for all their students during problem solving activities. There is initial evidence that computer-based feedback is effective for a subset of young students who struggle with mathematical equivalence (Fyfe & Rittle-Johnson, 2016).

Previous research indicates that elaborative feedback is vital for students who depend on operational patterns. When students receive only simple accuracy feedback, they might believe they only made a computation or procedural error rather than a conceptual error. For instance, a student solving  $6 + 2 + 8 = 5 + \underline{\quad}$  might add  $6 + 2$  to get 8, then add 8 more to reach 16, and finally add 5 to arrive at 21. As Hornburg (2017) found, when these students were told they used an incorrect strategy and encouraged to think of "another way" to solve the problem, they sometimes merely rearranged the numbers, such as calculating  $6 + 5 = 11$ ,  $2 + 8 = 10$ , and then  $10 + 11 = 21$ . Students who rely on operational patterns often require substantial scaffolding after an error to avoid reverting to these patterns in future problem-solving attempts.

Scaffolding through elaborative feedback can be provided as diminishing or accumulating over a problem set. Diminishing feedback provides students with the maximal level of support on the problem after making an error (i.e., step-by-step instructions for how to solve the problem) and then fades support on subsequent problems until students can correctly solve problems with only simple accuracy feedback. Accumulating feedback provides the support in the reverse order—beginning with simple accuracy feedback and then gradually increasing the support on subsequent incorrect problems until students reach the maximal level of support necessary to solve the problem correctly. Accumulating

feedback is typical of lab-based studies in the math equivalence literature, and it is based on the idea that it is beneficial to provide just enough support to promote discovery and growth but not so much that leads to passive, shallow learning (e.g., McDaniel & Schlager, 1990). Diminishing feedback is supported by learning theories that suggest that change resistance makes it difficult for students to shift from familiar but incorrect problem-solving strategies (McNeil & Alibali, 2005). Theories of change-resistance suggest that students may benefit from having problem-solving strategies modeled first, and then receiving gradually fading support.

### Developing and Implementing ICUE Online

The existing, paper-based version of the Improving Children's Understanding of Equivalence (ICUE) intervention supplements teachers' regular mathematics instruction to help students develop a robust, formal understanding of mathematical equivalence. The ICUE intervention was designed to decrease student activation of and reliance on operational patterns while simultaneously promoting a rich, formal understanding of mathematical equivalence. The intervention consists of four components (1) nontraditional arithmetic practice, (2) exposure to the equal sign in other contexts (e.g.,  $28 = 28$ ) before introducing arithmetic expressions within equations, (3) concreteness fading exercises that first present concrete, real-world, relational contexts (e.g., sharing stickers or balancing a scale) before fading into the corresponding abstract mathematical symbols (e.g., numerals, operators, the equal sign), and (4) activities that require students to compare and explain different problem formats and problem-solving strategies.

A previous efficacy study of the ICUE materials found that students in the intervention group outperformed students in the active control on proximal and transfer measures of equivalence with no observable tradeoffs in computational fluency (Davenport et al., 2023). The findings suggest the ICUE intervention helps students construct a robust understanding of mathematical equivalence. Despite the success of the intervention in improving overall rates of basic proficiency, many students fail to reach mastery. Given the pragmatic constraints of providing feedback on paper-based materials, teachers cannot provide individualized assistance in real-time. As students may be reinforcing incorrect operational patterns each time they make errors, a primary rationale for creating an online version of the ICUE intervention is the ability to provide just-in-time feedback to students.

During the iterative research and development of ICUE Online, we designed and tested different types of student feedback to determine which types are most effective for second grade students. As second graders are frequently not fluent readers and audio feedback is distracting in classrooms, we designed and tested feedback that is primarily visual.

## Present Study

The present study is designed to establish usability of ICUE Online and test the use of feedback with 2nd grade students. Our goal is to determine which types of feedback are most useful in helping students remedy operational misconceptions if they cannot be prevented. This research is a component of a larger project to iteratively develop and evaluate the ICUE Online materials. This answers the research question: **Is accumulating or diminishing feedback more effective at improving student mathematical outcomes for (a) equation problem solving and (b) conceptual understanding of the equal sign?**

## Methods

### Design

The design of the A/B test focused on evaluating the effectiveness of two feedback conditions—accumulating and diminishing—in fostering students' understanding of mathematical equivalence. We chose to use an A/B test to inform the design of ICUE Online as a first step prior to establishing its efficacy. Students were randomly assigned within class to one of the two feedback conditions in the final two lessons of ICUE Online. In the accumulating feedback condition, students received progressively more detailed and scaffolded cues; conversely, the diminishing feedback condition offered students less detailed and scaffolded cues across the problem set. The purpose of A/B testing is to directly compare competing theories of feedback design for mathematical equivalence problems, which will enable us to answer theoretically important questions about the effectiveness of our feedback approach in a more authentic context. The results of A/B testing will help determine whether limiting erroneous responses makes students more likely to develop correct conceptions about equivalence and problem solving.

### Procedure and Materials

Each teacher participated in a 30-minute online training. The training reviewed the study purpose and implementation guidelines. Teachers were instructed to (1) administer the pre-assessment prior to administering the ICUE online lessons (i.e., lessons 1-12), (2) assign lessons 1-12 in chronological order, (3) assign two 15-minute lessons per week for a total of six weeks, and (4) administer the posttest the week after the final lesson complete. The procedures for accumulating and diminishing conditions were identical, differing only in the order of the feedback in lessons 11 and 12.

When administering each lesson, teachers sent students a Qualtrics link where students entered a unique 3-word code as their identifier. Students worked independently or in groups; this was up to the discretion of the teacher. Each lesson was designed to take about 15 minutes to complete, and teachers could choose to stop or continue after the 15 minutes was up. Teachers were not asked to provide any

review to better understand how students are able to navigate computer-based problem sets independently.

Teachers were asked to complete a brief online teacher log for each week of implementation. Teachers reported which ICUE Online lessons they administered that week, how much time they spent preparing for and administering the ICUE Online lessons, and if their classrooms encountered any technical or content-related difficulties with ICUE Online for each log. Teachers were also asked to share any open-ended feedback about the study. Teachers reported spending approximately 28 minutes of classroom time per week (across two assignments), which aligns with how long the lessons were designed to take. The vast majority of teachers reported having no issues implementing ICUE online in their classroom, with only six teachers reporting an issue only once or twice across the seven weeks of the study. A subset of 3 teachers was selected for a classroom observation to further understand the feasibility of using the activities in the classroom. Aligning with teacher self-report from the implementation logs, no technological or implementation issues were observed.

**Accumulating and Diminishing Feedback** We designed five levels of scaffolded feedback based on the accumulating feedback previously used to help students construct better understandings of math equivalence (Hornburg, 2017). The specific features of these GIFs were carefully iterated through a series of usability tests (Bartel et al., 2023). The first level, accuracy feedback, informed students if they had gotten an answer incorrect with text that prompted the students to “Try again!”. The second level, perceptual support, increased the equal sign symbol’s salience by changing the color to blue and emphasizing it by a finger pointing at it. The third level, conceptual strategy, added the text “is equal to,” highlighting the equal sign as a relational symbol. The fourth level, analogy, added a concrete representation of the numbers in the equation as dominos. The fifth level, conceptual strategy plus explicit procedure, showed how to solve the problem by performing the operations on both sides of the equation. See Figure 1.

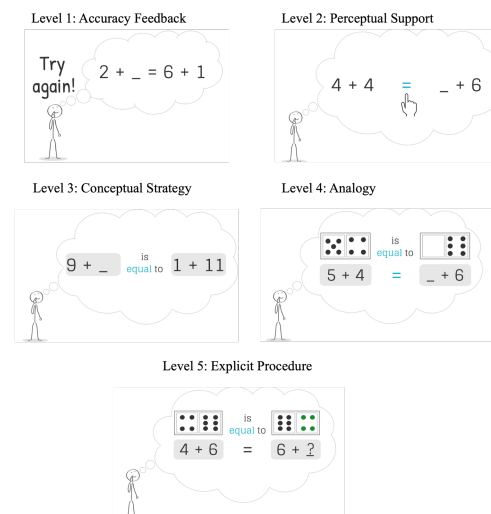


Figure 1: The five levels of accumulating and diminishing feedback. In the diminishing condition, students received the Level 5 GIF first. In the accumulating condition, students received the Level 1 GIF first.

Lessons 11 and 12 were the only two lessons that included accumulating or diminishing feedback. We chose to implement the feedback in lessons 11 and 12 because it was the point in the original paper and pencil ICUE curriculum where students are most likely to start making noticeable errors during the independent problem-solving component because of their reliance on operational patterns. The feedback was only presented to students in these lessons if they answered an item incorrectly on their first attempt. Students did not receive feedback if they answered the item incorrectly again on their second attempt. Lessons 11 and 12 contained 15 items each, and each of the five levels of feedback had the opportunity to be presented three times (i.e., In the accumulating condition, if students answered item one, two, or three incorrectly, then accuracy feedback was displayed).

**ICUE Online Lessons** Our study team adapted the first 12 lessons from the paper-and-pencil ICUE workbook into 12 ICUE Online lessons. The ICUE online materials expand on the early paper-and-pencil lessons, so students experience a natural progression of the materials that builds their knowledge. We condensed lessons 1-12 of the original ICUE workbook into 10 lessons for ICUE online. We developed two additional lessons to orient students to the nature of the items and feedback. Lessons 1-10 of ICUE Online contain items that provide extra scaffolding. These scaffolds include introducing the equal sign before any equation solving and concreteness fading exercises. Students solve non-traditional arithmetic problems in lessons 11 and 12. The non-traditional arithmetic problems include equations with operations on both sides of the equal sign and varied the position of the “answer” (e.g.,  $2 + 3 = 4 + \underline{\quad}$ ;  $2 + \underline{\quad} = 4 + 1$ ).

**Pre- and Posttest** To measure equation solving performance, students completed an online assessment with four free-response problems that had operations on both sides of the equation:  $8 + 2 = \underline{\quad} + 6$ ,  $7 + 4 + 6 = \underline{\quad} + 3$ ,  $3 + 7 = 3 + \underline{\quad}$ , and  $6 + 2 + 8 = 4 + \underline{\quad}$  (similar to McNeil & Alibali, 2005). To measure equal sign understanding, students responded to a set of questions about the equal sign. An arrow points to an equal sign presented at the top of the page and the text reads: (1) “What is the name of this math symbol?” and (2) “What does this math symbol mean?” Davenport et al. (2023) reported Cohen’s Kappa for Equal Sign Naming of 0.97 at pretest and 0.98 at posttest, and Defining the Equal Sign was 0.86 at pretest and 0.84 at posttest.

## Participants

The study was conducted across 10 elementary schools (5 public, 4 private, and 1 charter) and involved 17 teachers and

359 2nd grade students in (self-contained classrooms). None of the participating teachers had prior experience with ICUE. Of the student participants, 180 were part of the accumulating condition and 179 were part of the diminishing condition. The student sample included 183 boys, 173 girls, and 3 students for whom gender information was not reported. Three hundred students were categorized as non-English Language Learners while 59 were identified as English Language Learners. 195 students identified as White, 66 as Hispanic/Latino, 24 as Asian, 16 as Black, 2 as Native Hawaiian or Other Pacific Islander, 1 as American Indian or Alaska Native, and 30 as belonging to multiple categories.

## Results

We used a linear model to estimate the average impact of accumulating and diminishing feedback on student achievement (as measured by our researcher developed posttest). We included student pretest score, student gender, student race/ethnicity, and student English Language Learner (ELL) status as covariates. This is the same covariate structure we used for all of the following models. We decided to use a single-level model because of the student-level assignment. We used this model to address our research question: **Is accumulating or diminishing feedback more effective at improving student mathematical outcomes for (a) equation problem solving and (b) conceptual understanding of the equal sign?** We found, on average, that students in the accumulating ( $M = 2.55$ ,  $SD = 1.89$ ) condition scored higher on posttest than those in diminishing ( $M = 2.45$ ,  $SD = 1.85$ ) condition, but this difference was not statistically significant,  $F(1, 344) = .23$ ,  $p = .62$ , Hedges’  $g = .06$ . See Figure 2.

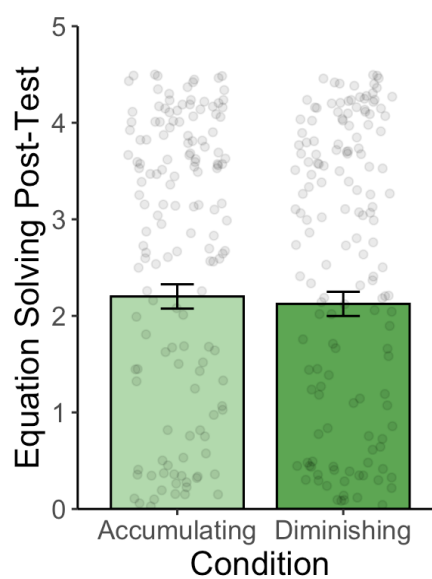


Figure 2: Equation solving unadjusted posttest score as a function of feedback condition (accumulating or diminishing).

While there was no significant difference between conditions, a paired samples t-test showed that students in the accumulating (gain score = 1) and diminishing feedback (gain score = 1.1) conditions both saw a significant increase in equation solving items from pre- to posttest,  $t(358) = 13.93, p < .001$ .

We investigated whether producing a relational definition of the equal sign (relational vs. not relational) at posttest differed as a function of receiving accumulating or diminishing feedback. To test this, we ran a generalized linear model (GLM) using the binomial family with the logit link function. We found that students in the accumulating ( $M = .34, SD = .47$ ) condition had a higher likelihood of producing a relational definition when defining the equal sign compared to those in diminishing ( $M = .32, SD = .46$ ) condition, but this difference was also not statistically significant,  $\chi^2(1) = .35, p = .49$ . Similar to above, while there was no statistical difference between conditions, the likelihood of producing a relational definition was significantly higher on post-test in the accumulating (an increase of .23) and diminishing conditions, respectively (an increase of .18).

In a recent exploratory analysis, Miyahara (2025) did an analysis of accumulating versus diminishing feedback from online symbolic equation solving problem sets inspired by the paper-and-pencil ICUE materials. She found that the effectiveness of accumulating versus diminishing feedback varied by student gender. We conducted an exploratory analysis examining the impact of accumulating versus diminishing feedback on the equation-solving posttest as a function of student gender. We used the same linear model above to test the interaction between feedback type and student gender on student achievement, replacing the feedback term with an interaction term. There was a significant interaction between feedback type and student gender,  $F(1, 343) = 3.95, p = .04$ . The pattern suggests boys performed equally well on posttest regardless of whether they received accumulating or diminishing feedback, while girls who received accumulating feedback performed better than those who received diminishing feedback. See Figure 3.

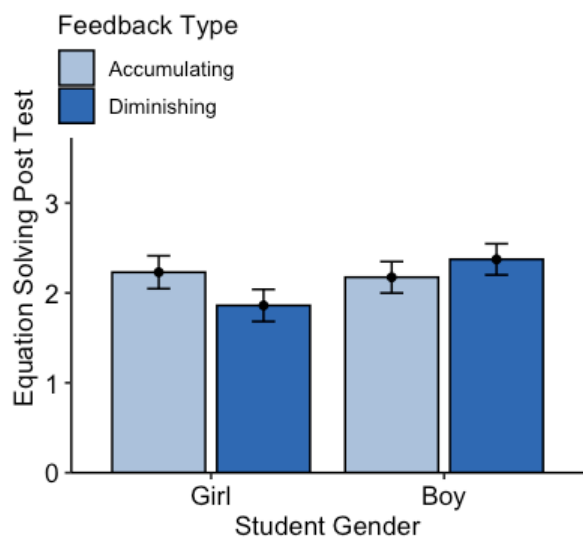


Figure 3: Equation solving posttest score as a function of student gender (girl or boy) and feedback type (accumulating or diminishing).

## Discussion

This study explored the effectiveness of accumulating and diminishing feedback in enhancing elementary students' understanding of mathematical equivalence, a crucial skill for future success in algebra. We also aimed to develop preliminary evidence of ICUE Online's efficacy and whether it could improve students' equation solving like the paper-and-pencil version does. The study found that both feedback groups improved in equation solving and conceptual understanding of the equal sign from pre- to posttest. However, we did not observe a statistically significant difference between the two feedback conditions. This finding may suggest the value of immediate feedback in addressing entrenched operational patterns and highlights the potential of both feedback methods to enhance students' skills. Further, observing gains in conceptual understanding is noteworthy, as it can be challenging to disrupt operational patterns of thinking. These positive changes resulted from a modest intervention compared to the original paper and pencil version of ICUE (e.g., 32 lessons across 16 weeks)—on average, teachers implemented the ICUE online materials for 28 minutes per week over seven weeks—which effectively improved students' conceptual understanding of the equal sign and equation-solving performance. This demonstrates the potential impact of even limited, well-designed interventions when integrated into the curriculum.

An exploratory analysis revealed a significant interaction between feedback type and student gender, where girls benefited more from accumulating feedback, while boys' performance remained consistent across conditions. Miyahara (2025) found a different pattern of results: while gender similarly moderated the effects of feedback in growth, girls benefitted more in the diminishing condition and boys benefited more in the accumulating condition. However, the setting and exposure to the ICUE materials in the present study and Miyahara (2025) differed significantly. Miyahara (2025) tested accumulating and diminishing feedback in the context of a broader tutoring program. Independent of their research, all students in the tutoring program receive the same pre- and posttest used in the present study as a progress monitoring measure, once at the beginning of the semester and once at the end of the semester. During two tutoring sessions within the semester, tutors administered two online problem sets adapted from the original paper and pencil ICUE intervention. All students received the same first problem set aimed to activate children's informal understanding of equivalence. In the second problem set, students were randomly assigned to solve symbolic equations while receiving either diminishing or accumulating feedback. Additionally, in Miyahara (2025), students were only exposed to diminishing or accumulating feedback in the second lesson. All students received the same first lesson,

consisting of 29 multiple-choice questions focusing on providing concrete examples of mathematical equivalence to activate children's informal understanding of equivalence. In contrast, students in the present study spent several sessions across weeks to build the background knowledge theorized to help students prepare for lessons 11 and 12.

Prior research suggests that girls are more likely to use taught procedures (Carr & Jessup, 1997, Carr et al., 1999). One can theorize that in Miyahara (2025)'s study, when the exposure to the ICUE materials were minimal prior to equation solving with feedback, the girls benefitted from being exposed to the most scaffolded cue right away. However, in the present study, the students had gradual scaffolding across 10 lessons prior to solving the more difficult lessons 11 and 12. In other words, the girls benefitted from the explicit procedures and foundation in the earlier lessons, leading them to eventually perform better with a less scaffolded cue. In Miyahara's (2025) study, the tutoring students were quickly exposed to solving the difficult symbolic equations. In the present study, we designed the ICUE online materials to build the requisite conceptual knowledge prior to being exposed to the more difficult equation solving items.

The success of incorporating immediate feedback into the ICUE online lessons is promising for their continued development. However, one limitation of this study is the lack of a control condition. The purpose of this study was to provide initial evidence for whether accumulating *or* diminishing feedback was most effective rather than testing whether either were better than no feedback at all. The next step for development of the ICUE online materials is to test the accumulating feedback condition against a control condition. This will answer open questions regarding whether it was actually the feedback that led to gains on the posttest, or if these gains can merely be attributed to extra mathematical practice across several weeks.

The present study may suggest the importance of immediate, elaborative feedback in helping students grasp mathematical equivalence. Given that teachers often struggle to provide real-time, individualized feedback, technology-based interventions like ICUE Online can supplement classroom instruction by delivering timely, automated support. Although overall differences between accumulating and diminishing feedback were not significant, the observed interaction with student gender suggests that tailoring feedback to individual learner characteristics may enhance effectiveness. Future research should explore whether personalized feedback adaptations improve outcomes, particularly for students at risk of persistent misconceptions. The present study is a small steppingstone toward advancing the development of effective instructional interventions for early mathematics education that can be delivered at scale. By refining automated feedback and addressing student-specific learning needs, educators and researchers can strengthen foundational mathematical understanding and better prepare students for future algebraic success.

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