Continuity and Discontinuity in Children's Number Acquisition

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Abstract

Recent research has suggested that some children, after they learn the meaning of "four," proceed to learn the next few numbers one at a time (Krajcsi & Fintor, 2023). This claim is in direct contrast with previous theories that argued for an inductive leap after learning "four" (Carey, 2009). We assessed children's number knowledge using an adapted Give-N method that captured higher set sizes, and tests of executive function and working memory as an exploratory window onto the changes across stages. First, results support the claim that children exhibit partial-number knowledge beyond "four". Second, executive function was associated with children's jump from knowing numbers up to "four" (subset-knower stages) to knowing numbers above four, while working memory was associated with the change from partial to putatively full knowledge of number word meanings. These findings offer new insight into the conceptual change process and suggest a potential two-fold sequence in children's induction of cardinality.

Keywords: number; number acquisition; conceptual change; learning mechanisms

Introduction

Research on numerical cognition is largely divided between conceptual theories emphasizing continuity versus discontinuity in the acquisition of cardinality: the idea that the last word in a counted set represents the total value of the set. The most well-known articulation of the discontinuity hypothesis is Carey's bootstrapping theory (Carey, 2004; Sarnecka & Carey, 2008). To date, the Give-N task, often considered the gold-standard measure of understanding cardinality, has been used to suggest that children go through an inductive leap after learning the meanings of the numbers three or four (Wynn, 1992). Specifically, children learn the numbers one, two, three, and four in a slow and piecemeal fashion, and then generalize their understanding to higher numbers.

However, this perspective has been challenged in two ways: in conceptualizations arguing that an understanding of cardinality is present long before children show this skill on Give-N (Gelman & Gallistel, 1978), and in more recent evidence that children's induction of the logic of counting is neither as sudden nor as complete as early versions of the bootstrapping theory would suggest (e.g., Davidson, Eng, & Barner, 2012).

Most recently, Krajcsi and Fintor (2023) argued that the principle of cardinality is acquired gradually. According to them, the standard Give-N protocols do not assess numbers beyond five, resulting in an overestimate of children's knowledge: children are designated 'cardinal-principle knowers' once they show knowledge of five or six, but it is possible that children's knowledge is limited above those set sizes. To assess this theory, they administered a new method of the Give-N task asking participants to produce set-sizes of 1, 2, 3, 4, 5, 6, 7, 8, and 9 in a pseudo-random order, for a total of 4 blocks or 36 trials (Krajcsi & Fintor, 2023). They found evidence of five- and six-knowers, who were clearly beyond the level of four-knowers, but did not meet standard classification criteria for CP-knower or subset-knower levels. Referring to them as "Large Subset Number Knowers" or LNS-knowers, this new approach to classifying early number knowledge invites a revised conceptualization of the progression to cardinality.

Specifically, this new hypothesis offers critical theoretical questions surrounding the cognitive mechanisms that underlie conceptual change. The main account of discontinuity in number development posits a discrete change around or after the acquisition of "four." The key idea is that the meanings of "one", "two," and "three" are based in an innate cognitive system, the object file system, which exhibits a set size limit and therefore cannot be used to underpin the meanings of larger quantities (Carey & Barner, 2019). Therefore, a different learning mechanism, such as bootstrapping, is needed to acquire meanings of larger numbers, inducing the meanings of numbers above four from the patterns observed in the first few numbers. Theorists who favor continuity argue that the acquisition of cardinality does not involve a semantic induction about the structure of the count list, but rather is a more gradual process, for example via improved mappings to or acuity of the approximate number system or ANS (Szkudlarek & Brannon, 2017) or gradual integration of the counting principles (Davidson et

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al., 2012; Cheung et al., 2022).

Inspired by the rich debate and by Kracjsci & Fintor's suggestion, which we agreed with, that the possibility of higher knower levels had not been adequately considered, we investigated previously collected data and ran a new study to test the LNS-hypothesis.

First, using the criteria presented by Krajcsi & Fintor, we re-analyzed Give-N (n= 475) data that was collected for a larger intervention study. In this study, we used the titration method for Give-N with the distinction that instead of titrating with the number 6 when children correctly responded to the number 5, they instead received one trial each for the numbers 6, 7, and 8 asked out of order (8 before 7). Children were originally classified as CP-knowers (n = 229 out of 475) if they were correct on two-thirds of the trials for 6, 7, and 8 collectively. In our re-coding, we categorized children as potential 6 or 7 knowers if they were correct up to that set size but not higher. We found evidence that 8.2% (19/229) demonstrated partial number knowledge without correctly responding to all three trials with set size 6, 7, and 8 (a similar rate was reported in Gordon et al., 2021). The results provided preliminary evidence that some children might know number words for larger set sizes, or a subset of their count list, without fully generalizing to the cardinal principle.

These initial findings suggest a need to take a new look at discontinuity theories of number acquisition. It is likely that previous studies have failed to find 5-, 6-, and 7-knowers because they failed to look for them carefully. Thus, it is possible that children progress through more knower-levels than previously assumed, but that the LNS stage is fleeting, explaining why relatively few children in any dataset demonstrate the pattern. It is also possible that children's number acquisition is more variable than previously acknowledged (i.e., if only some children exhibit the LNS-knower stages). Either way, understanding the LNS knowers is critical to a full understanding of the development of number concepts, as they represent an intermediate stage between small-number subset (SNS) knowers and full integration of cardinality. A central question is how LNS knowers learn the meanings of the number words above "four" and whether they have made an induction about counting and cardinality as hypothesized by Carey's bootstrapping hypothesis.

To address this question, we turned to recent research on the distinction between *conceptual construction* and *knowledge enrichment*. Several studies suggest that executive function is implicated in building theoretical and conceptual knowledge, but not necessarily for learning facts (Bascandziev et al., 2018; Tardiff et al., 2020; Zaitchik et al., 2014). For example, in the domain of biology, learning that crickets have ears on their legs represents knowledge enrichment that happens quickly and easily; by contrast learning that a stuffed animal is not alive requires developing a new, vitalistic theory about what differentiates life from non-life states. Understanding a statement like 'the heart pumps blood that carries oxygen through the body' requires a much more laborious process of building a coherent theory about how different parts of the body work together to maintain life. In particular, two components of executive function, inhibition and switching, but not working memory, have been implicated in conceptual construction in biology (Tardiff et al., 2020). By analogy, if learning the meanings of number words beyond "four" requires conceptual construction, executive function might be involved more so than working memory. On the other hand, if learning number meanings beyond "four" requires procedural counting skills but does not entail conceptual construction, then EF abilities like inhibition and switching might not be involved. Instead, measures like working memory might be more predictive.

Crucially, EF and working memory, as well as other domain-general capacities like receptive vocabulary, have been correlated with number acquisition (Negen & Sarnecka, 2012; Gordon et al., 2021; Emslander & Scherer, 2022). Furthermore, working memory is clearly required to enact the counting procedure. It is thus possible that working memory fully explains the jump from SNS to LNS knowledge.

The question, then, is whether EF or WM measures better differentiate large-number subset (LNS) knowers from small-number subset (SNS) knowers, after controlling for age. If LNS knowers are simply children who have made no conceptual leaps but are just more skilled procedural counters than earlier knower-level stages, we might expect LNS-knowers to have higher working memory than SNS-knowers, but not necessarily higher abilities in other EF tasks. Alternatively, if LNS-knowers have embarked on the difficult process of conceptually constructing natural number concepts above four, then we might expect LNS-knowers to have higher EF than SNS-knowers, but not necessarily higher working memory.

Given that LNS-knowers are a newly recognized phenomenon, it is not clear which explanation for their performance is correct nor how to revise theories of number acquisition in light of them. As a first look to understand this conceptual stage, we ran an exploratory study to more rigorously probe for the existence of LNS-knowers and to test whether EF, WM, or both kinds of domain-general abilities differentiate LNS-knowers from other knower-levels, specifically classic subset-knowers, i.e., the small-subset-knowers, and classic cardinal-principle (CP) knowers.

In addition to differentiating LNS- from SNS-knowers, it is also worth considering how they might differ from CP-knowers. We note that our measure of "knowing CP" here simply means that children passed at the highest set size we asked for, namely 8. It is possible that, had we asked up to 9, we would have found 8-knowers, and had we asked up to 10, we would have found 9-knowers. For purposes of simplicity and consistency with the existing literature, we call all children who passed at all set sizes 'CP-knowers', though we acknowledge that we lack definitive evidence about how high they can go in engaging meaningful counting. We then ask in what ways, if any, LNS knowers differ from CP-knowers. Again, if EF differentiates these two groups, we might infer that the conceptual construction comes only after learning four or five, casting doubt on previous theories about the point of change. If WM differentiates LNS from CP-knowers, then LNS knowers' abilities might be better explained as a difficulty with holding on to the complex procedures of counting rather than by failure to undergo conceptual change.

The Present Study

The existence of large-number subset (LNS) knowers raises important theoretical questions about the process of number acquisition. Therefore, this study aims to answer two fundamental questions: 1) Using a modified Give-N protocol, do we find evidence of partial number knowledge – children who successfully demonstrate stable knowledge of number words meanings for 5, 6, or 7, but not cardinal principal knowledge (i.e., the large-number subset-knower (LNS) hypothesis)? and 2) If LNS-knowers are found, is their performance systematically associated with domain-general skills like executive function and working memory, differentiating them from SNS and CP knowers?

We hypothesize that, when using an adapted Give-N assessment, we will find evidence of large-number subset knowers – in other words, some children will demonstrate stable knowledge of, and produce sets for, the number words up to 5, 6, and/or 7, but fail to consistently produce sets for all of the larger numbers (e.g., a child will know "six" but not "seven" and "eight"). We also expect to find a relationship with domain-general skills that can explain their success with larger set sizes but failure to demonstrate full knowledge of cardinality.

Methods

Participants

A total of 144 3- and 4-year-old children attending preschool (M_{age} = 53 months, SD = 5.93 months; range = 39.2 – 66.3 months) participated in the study. Children were tested on 5 measures across two sessions, though the current study focuses only on the three measures administered during the first testing session. An additional 3 children were tested but did not complete the assessments.

Preschool classrooms in Central CT were recruited via phone calls and emails to school directors. The study was conducted at the schools and all children had a signed parental consent form for children's participation. No demographic information was collected, but children attended a range of public and private schools in low-, middle-, and high-income settings.

Design and Procedure

All children completed three tasks in a fixed order in one testing session.

Give-N (Wynn, 1990; 1992) Children completed an adapted version of the Give-N task. The stimuli included 15 plastic fish and a blue bowl (Wynn, 1992). Experimenters presented the blue bowl as a "swimming pond" for the fish and asked the participants to place the requested number of fish in the bowl (e.g. "Can you put N fish in their swimming pond?"). Irrespective of response, testers always asked the children to count and check ("e.g., "Is that N fish? Let's count and check!") and provide them the opportunity to correct their response. All children started with the same first two trials, one asking for 4 and one asking for 3. If children were correct on both trials, they were asked for 4 again. If children were incorrect on either of the first two trials, titration proceeded backwards until the experimenter determined the highest number 1 through 4 that the child knew, using a standard titration method.

Children who were designated as "4-knowers" in the first part of this procedure then moved into the adapted Give-N to probe for LNS-knowers. They received 3 blocks of trials asking for sets of 5, 6, 7, and 8, in a fixed, pseudo-random order. Children followed the "count and check" routine after each set size. All children received these 12 trials irrespective of correct responses. Children were considered an N-knower using the same criteria traditionally used to categorize small-number subset knowers. That is, if they correctly gave N objects on 2 out of 3 trials on which they were asked for N and avoided giving N on at least two-thirds of trials that ask for more than N, they were considered an N-knower. Children were considered a cardinal-principle (CP) knower when they succeeded at responding correctly to all set sizes they were asked for (in this case, up to "8").

Head-Toes-Knees-Shoulder (HTKS; McClelland et al., 2014) The HTKS task is a behavioral regulation task made up of three parts used to test executive function. In the first part, participants were asked to touch their heads in response to a prompt to touch their toes, and to touch their toes in response to a prompt to touch their heads. In part two, a second rule was added where participants were asked to also remember to touch their shoulders in response to a prompt for knees, or to touch knees in response to a prompt for shoulders. Finally, in part three, the rules were switched so that head-knees and toes-shoulders became new paired opposites. Each trial was scored with a 0 for incorrect responses, a 1 for self-correction (i.e., initial movement to command was incorrect but child self-corrected) and 2 for correct responses. Each of the three parts was worth a maximum of 20 points, making 60 the highest possible score. Participants had to achieve a minimum score of 4 on a given part to advance to the next section.

Picture Memory (PM; Weschler, 2012) To assess working memory, children were given the Picture Memory task from the Wechsler Preschool and Primary Scale of Intelligence-Fourth Edition (WPPSI-IV; Wechsler, 2012). Participants were shown a set of images for a set amount of time (e.g., 3 or 5 seconds), based on their age. Children were then asked to select those same pictures from the options provided on the response page. Responses were coded as correct only if the children correctly recalled all the images they saw on the previous page. The assessment ended upon completion of all trials or after the child responded incorrectly on three consecutive items.

Results

Descriptives

Most of the participants were able to be categorized into knower-levels using the standard coding (n=133; Table 1). However, there were an additional 11 participants that did not fit the traditional coding patterns for Give-N (i.e., they were incorrect on two-thirds of trials for N+1 but then correct on at least two-thirds of trials for N+2 and/or N+3). We categorized these children as "skip-knowers" and provide descriptive information about them here; they are otherwise not analyzed here, and we return to them in the discussion.

Table 1: Frequency of children at each knower-level

Knower Level	Frequency (%)
Pre	3 (2.1%)
1	3 (2.1%)
2	4 (2.8%)
3	5 (3.5%)
4	5 (3.5%)
5	2 (1.4%)
6	4 (2.8%)
7	5 (3.5%)
CP (8)	102 (70.8)%
Skip	11 (7.6%)
Total	144 (100%)

We then split the participants into three categories by knower-level (small-number subset knowers, SNS; large-number subset knowers, LNS; and cardinal-principle knowers, CP). Descriptive data for age, EF, and WM in each category are displayed in Table 2.

Table 2: Mean (SD) of age, EF, and WM scores by knower-level

	Age	HTKS	PM
SNS	48.60 (6.93)	4.65 (7.32)	5.00 (3.38)
LNS	49.18 (5.08)	14.73 (12.73)	5.91 (3.78)
CP	56.29 (4.69)	25.51 (17.43)	8.67 (4.18)

Next, we ran partial correlations between our three outcomes (binned knower-level, HTKS, and PM) controlling for age (see Table 3), and found that both EF and WM were significantly correlated with knower-level category.

Table 3: Partial correlations controlling for age

	Knower-level	HTKS
HTKS	0.315^{***}	—
PM	0.361^{***}	0.183^{*}

Knower-level uses Spearman's rho*; $^{*}p < .05$, $^{***}p < .001$.*

Associations of executive function and working memory with knower-level status

To examine the effect of executive function (EF) and working memory (WM) on children's knower-level status, we next ran a series of logistic regressions with knower-level status (SNS, LNS, or CP) as the outcome of interest, after controlling for age at time of test. The overall model was significant (X^2 (3) = 67.7, $p < 0.001$, $R^2_{\text{McF}} = 0.39$) suggesting that the model was effective at differentiating between different knower-level categories. All three predictors were statistically significant (see Table 4), suggesting that age, EF, and WM are all associated with children's progression through the knower-level stages.

Table 4: Ordinal logistic regression predicting knower-level

					95% confidence interval	
Predictor	Estimate	SE	Z	Odds ratio	Lower	Upper
Age	0.243^{***}	0.06	4.2	1.27	1.15	1.44
HTKS	0.060^{**}	0.02	2.67	1.06	1.02	1.12
PM	0.274^{***}	0.08	3.42	1.32	1.14	1.56

$^{**}p < .01$, $^{***}p < .001$

We next took up the question of whether LNS-knowers are more categorically similar or different to SNS knowers or CP-knowers. We therefore ran two binary logistic regressions, the first looking at whether age, EF, and WM would predict LNS vs SNS status, and the second predicting LNS vs CP status. In our first model predicting SNS vs LNS status, the model was significant overall (X^2 (3) = 7.86, $p <=$ 0.049). EF was a statistically significant predictor of SNS vs. LNS level, but neither age nor WM were statistically significant predictors of SNS vs. LNS level (Table 5). Thus, executive function, but not working memory, differentiates LNS from SNS knowers.

Next, we tested whether LNS vs CP-knower status was significantly predicted by the same variables. The overall model was again significant (X^2 (3) = 27.5, p < = 0.001), but in contrast to the SNS-LNS model, we found no significant effect of EF on knower-level status. EF did not predict whether a child was an LNS or CP-knower. Both age and WM significantly predicted children's knower-level status (Table 6). Thus, working memory, but not executive function, differentiates LNS knowers from CP knowers.

Table 5. Binary logistic regression predicting SNS vs LNS status

					95% confidence interval	
Predictor	Estimate	SE	Z	Odds ratio	Lower	Upper
Intercept	0.963	4.120	0.234	2.62	8.16E-04	8411.735
Age	0.029	0.087	0.333	1.03	0.868	1.221
HTKS	-0.128*	0.062	-2.049	0.88	0.779	0.994
PM	-0.144	0.131	-1.099	0.866	0.67	1.119

Note. Estimates represent the log odds of "SNS" vs. "LNS", * $p < .05$

Table 6. Binary logistic regression predicting LNS vs CP status

					95% confidence interval	
Predictor	Estimate	SE	Z	Odds ratio	Lower	Upper
Intercept	-19.164***	5.883	-3.258	4.76E-09	4.68E-14	4.84E-04
Age	0.354***	0.106	3.346	1.43	1.158	1.75
HTKS	0.021	0.026	0.779	1.02	0.969	1.08
PM	0.316*	0.128	2.469	1.37	1.067	1.76

Note. Estimates represent the log odds of "LNS" vs. "CP", *** $p < .001$

Discussion

The current study systematically tested the possibility that some children reliably demonstrate partial knowledge above the "four"-knower stage but below the standard of a cardinal-principle knower – children dubbed "Large Number Subset" or LNS-knowers by Krajcsi & Fintor (2023). The results provide preliminary evidence that some children do indeed follow this pattern, passing the criteria for knowing up to five, six, or seven, but not eight. These results suggest that, contrary to strict discontinuity theories of number acquisition, the process of gaining number word meanings may be more gradual, with additional stages between "four"-knowers and CP-knowers. The relatively small number of these children tempers this argument somewhat–it is not clear from these cross-sectional data whether a brief LNS-knower stage is typical of most children as they come to understand cardinality, or whether this is an unusual pattern, for example of some children who have difficulty extracting the add-one pattern that organizes counting. Further analyses, like the exploration of whether there are meaningful differences in EF and WM between skip-knowers and LNS-knowers, who both demonstrate incomplete knowledge of numbers beyond 4, are needed with larger samples.

To address the question of what is learned in each step – going from small-number subset knower to large-number subset (LNS) knower, and from LNS-knower to cardinal-principle (CP) knower, we added assessments of working memory and executive function. We drew on prior theory and evidence indicating that executive function is more related to conceptual change while working memory is more related to knowledge accumulation (Bascandziev et al., 2018; Tardiff et al., 2020). We initially hypothesized that LNS-knowers might be children who struggled to extract a pattern or generate a new theory of numbers and consequently theorized that they might have low executive function relative to CP-knowers. Further, we theorized that they might outperform subset-knowers on the procedural aspects of counting due to high working memory. In other words, we speculated that LNS-knowers would be best characterized as subset-knowers who had failed to generalize the successor idea of counting.

The results of our domain general tasks suggest something different. The first shift, from subset- to LNS-knower, was associated with significantly higher executive function, after controlling for age and working memory. The second shift, from LNS- to CP-knower, was associated with significantly higher working memory, after controlling for age and EF. If EF is an indicator of and catalyst for conceptual construction, then these results support the idea that going from subset- to LNS-knower represents a qualitative shift in reasoning, while going from LNS- to CP-knower represents elaboration or enrichment of an existing conceptual stage.

The evidence that LNS-knowers differ from subset-knowers in executive function is consistent with the claim that these two stages are qualitatively different. Researchers have made both logical and empirical arguments that the nature of learning number meanings above four must be different – and therefore discontinuous – from the mechanisms that drive learning for one to four (e.g., LeCorre & Carey, 2007). Most critically, they have argued that the object-file system provides the referents for small number word meanings, and the set sizes that can be represented by the object-file system are limited to about four. Potentially, LNS-knowers have made a first inductive leap in the organization of number word meanings: tying them to their role in the count list rather than representations of sets of objects. However, this leap seems to be limited.

In the second putative leap, LNS-knowers extend known number word meanings up to and beyond "eight." The limited generalization of LNS knowers suggests that getting an insight about cardinality is not sufficient to crystallize and apply it to higher ranges (see also Davidson et al., 2012). What function might working memory serve in the LNS- to CP-knower transition? Perhaps WM is needed to keep track of the counting procedure at higher numbers, or to coordinate other counting principles, like one-to-one correspondence, with cardinality (see also Cheung et al., 2022). Another possibility is that, once children have made the leap to fixing number word meanings by their place in the count list rather than by a previous representational system such as object-files, working memory is needed simply to keep building new number word meanings within this system, accumulating 'knowledge' within the newly acquired theoretical structure of natural number.

An important limitation is that children in this study were only tested up to "eight," and therefore the extent of the LNS-knower stage is unknown. Here, we refer to children who passed the trials for "eight" as CP-knowers, but their full knowledge of CP was not tested. It is possible that many more children, when carefully tested, bump up against a maximum number for which they have developed a meaning and can apply the counting procedure, and above which they systematically fail. As many researchers have pointed out, a better and more precise characterization of what it means to 'know' cardinality would greatly benefit the field.

A second limitation is that executive function and working memory are overlapping and complex concepts. Here we particularly emphasize the aspects of executive function – inhibition and switching – that have been theorized as potentially important for conceptual change, and we used a task that focuses on those elements. We note the relatively low (though statistically significant) correlation between our EF and WM tasks, which suggests that they tapped different abilities.

The current findings provide further evidence that support the claims of a new kind of knower-level stage, the large-number subset or LNS knower. These results force a revision in narratives of number acquisition that emphasize radical discontinuity. Drawing on the difference in EF scores, the data raise the strong possibility that this stage is qualitatively different from classic subset-knowers with meanings up to "four." The LNS-knowers might represent how learning looks right after the construction of a new intuitive theory, such as the system of natural numbers for articulating number word meanings.

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References

Bascandziev, I., Tardiff, N., Zaitchik, D., & Carey, S. (2018). The role of domain-general cognitive resources in children's construction of a vitalist theory of biology. *Cognitive Psychology, 104*, 1-28. doi:10.1016/j.cogpsych.2018.03.002

Carey, S. (2004). Bootstrapping & the origin of concepts. *Daedalus*, *133*(1), 59–68. 10.1162/001152604772746701

Carey, S. (2009). The Origin of Concepts. Oxford University Press. doi:10.1093/acprof:oso/9780195367638.001.0001

Carey, S., & Barner, D. (2019). Ontogenic origins of human integer representations. *Trends in Cognitive Science*, *23*(10), 823–835. doi:10.1016/j.tics.2019.07.004

Cheung, P., Toomey, M., Jiang, Y. H., Stoop, T. B., & Shusterman, A. (2021). Acquisition of the counting principles during the subset-knower stages: Insights from children's errors. *Developmental Science*, *25*(4), 1–12. Wiley Online Library. doi:10.1111/desc.13219

Davidson, K., Eng, K, & Barner, D. (2012). Does learning to count involve a semantic induction? *Cognition, 123*, 162–173. doi:10.1016/j.cognition.2011.12.013

Emslander, V., & Scherer, R. (2022). The relation between executive functions and math intelligence in preschool children: A systematic review and meta-analysis. *Psychological Bulletin*, *148*(5-6), 337. doi:10.1037/bul0000369

Gelman, R., & Gallistel, C. R. (1978). The Child's Understanding of Number. Cambridge, MA: Harvard University Press.

Krajcsi, A. & Fintor, E. (2023). A refined description of initial symbolic number acquisition. *Cognitive Development, 65*, 1–16. doi:10.1016/j.cogdev.2022.101288

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: an investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*(2), 395–438. doi:10.1016/j.cognition.2006.10.005

McClelland, M. M., Cameron, C. E., Duncan, R., Bowles, R. P., Acock, A. C., Miao, A., & Pratt, M. E. (2014). Predictors of early growth in academic achievement: The head-toes-knees-shoulders task. *Frontiers in psychology*, *5*, 599. doi:10.3389/fpsyg.201400599

Negen, J., & Sarnecka, B. W. (2012). Number-concept acquisition and general vocabulary development. *Child Development*, *83*(6), 2019-2027. doi:10.1111/j.1467-8624.2012.01815.x

Sarnecka, B. W & Carey, S. (2008). How counting represents number: what children must learn and when they learn it. *Cognition*, *108*(3), 662–674. doi:10.1016/j.cognition.2008.05.007

Gordon, R., Scalise, N. R., & Ramani, G. B. (2021). Give yourself a hand: The role of gesture and working memory in preschoolers' numerical knowledge. *Journal of Experimental Child Psychology*, *208*, 105145. doi:10.1016/jecp.2021.105145

Szkudlarek, E., & Brannon, E. M. (2017). Does the approximate number system serve as a foundation for symbolic mathematics? *Language Learning and Development*, *13*(2), 171–190. doi:10.1080/15475441.2016.1263573

Tardiff, N., Bascandziev, I., Carey, S., & Zaitchik, D. (2020). Specifying the domain-general resources that contribute to conceptual construction: Evidence from the child's acquisition of vitalist biology. *Cognition*, *195*, 1–12. doi:10.1016/j.cognition.2019.104090.

Wechsler, D. (2012). Wechsler preschool and primary scale of intelligence technical and interpretative manual (4th ed.). Bloomington: Pearson.

Wynn, K. (1990). Children's understanding of counting. *Cognition*, *36*(2), 155-193. doi:10.1016/0010-0277(90)90003-3

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, *24*(2), 220-251. doi:10.1016/0010-0285(92)90008-P

Zaitchik, D., Iqbal, Y., & Carey, S. (2014). The effect of executive function on biological reasoning in young children: An individual differences study. *Child Development*, *86*(1), 160–175. doi:19.1111/cdev.12145