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Calculation is a form of professional numeracy that, from the first evidence of writing in Egypt to the period of the New Kingdom, comprised concepts and methods different from those of contemporary Mesopotamia. In approximately the late fourth millennium BCE, professional numeracy developed in both regions. A comparison of these two early mathematical cultures allows insights into the parallel necessities that prompted this development, as well as elucidation of the differences in the formation of the two mathematical systems, their historiographical treatment, and the consequences of the choices of writing material for the preservation of sources.

عُرف الحساب على مستوى احترافي - مهني في مصر القديمة منذ ظهور الإرهاصات الأولى للكتابة. وحتى عصر الدولة الحديثة كان يتضمن مفاهيم وأساليب تختلف عن تلك السائدة في بلاد ما بين النهرين المعاصرة. وبحدود أواخر الألفية الرابعة قبل الميلاد تقريبًا، تطور الحساب الاحترافي المهني في كلتا المنطقتين. تتيح مقارنة هاتين الثقافتين الرياضيتين المبكرتين فهمًا للضرورات المتوازية التي دفعت إلى تطورهما، والاختلافات في تشكيلهما، وطرق تفسيرهما من قبل الباحثون، بالإضافة إلى تأثير اختيار مادة الكتابة في حفظ مصادرهما.

Egypt is among the earliest cultures of which written evidence of professional numeracy is extant. Depending on the focus of modern researchers and their stance on premodern knowledge, the realm of numeracy is termed "calculation" (*Rechnen*) or "mathematics" (*Mathematik*). While the aim of the procedures and techniques that were developed in this domain was to determine numbers or quantities for specific problems, the written documentation of those procedures and techniques (which may have originated from the context of teaching) indicates that they comprised more than simple arithmetic. In fact, actual calculation techniques—namely, those of multiplication and division, addition and subtraction—were not recorded in writing and are only found in the documentation of some of the mathematical problems. While the documentation focuses on the procedures for solving such mathematical problems, the style of presentation in the form of exemplary procedures probably contributed to the modern classification of the surviving mathematical texts as evidence for arithmetic, rather than as collections of mathematical knowledge. At approximately the same time that professional numeracy developed in Egypt (around the end of the fourth millennium BCE), it also developed in Mesopotamia.

History of Research

Research on ancient Egyptian numbers and arithmetic began in the second half of the nineteenth century and was initially based on the field calculations found on the walls of the Edfu Temple (Brugsch 1865). The discovery of the Rhind Mathematical Papyrus and its acquisition by the British Museum prompted the beginning of more substantial research on ancient Egyptian mathematics. This was undertaken by scholars from two disciplines, Egyptology and the history of mathematics, which explains the publication of two editions of the papyrus (Peet 1923 and Chace, Bull, and Manning 1927, 1929) within a few years of each other. (The historiography of Mesopotamian mathematics followed a similar development: cf. Høyrup 1996.) The few additional mathematical papyri that have been found were subsequently published by the first half of the twentieth century (cf. Griffith 1898; Schack-Schackenburg 1900, 1902; and Struve 1930; for further mathematical texts and later editions, cf. Imhausen 2016). Egypt, as well as Mesopotamia, is also included regularly in popular scientific works on ancient numeracy (cf. Georges Ifrah 1981); these often underestimate the complexity of the sources and include simplified and sometimes incorrect descriptions.

Apart from the editions of mathematical texts, several monographs on ancient Egyptian mathematics were published during the twentieth century, which often presented modern interpretations of the mathematical knowledge found in the sources, e.g., rendering the procedure texts as algebraic equations (Gillings 1972 and Couchoud 1993). The rendering of Egyptian calculation techniques through the lens of modern mathematics sometimes resulted in a negative assessment of the capability and efficiency of Egyptian mathematics, especially when compared to Mesopotamian mathematics; cast in this negative light, Egyptian mathematics were sometimes even accused of hindering the development of substantial astronomical knowledge in ancient Egypt (Neugebauer 1975: 559). With the fairly recent introduction of a method of comparing the sequences of steps in procedures (Ritter 1989, 2004), a more adequate strategy for analyzing procedure texts has become available that enables a modern reader to better assess the Egyptian concepts of organizing mathematical knowledge. This method has been used for Mesopotamian astronomical and mathe-matical texts (Ossendrijver 2012; Ritter 2010), as well as for Egyptian hieratic mathematical texts (Imhausen 2003), and has also enabled a comparison between them (Melville 2004).

While many researchers have focused on mathematical problem texts, which constitute the majority of the available sources, others have focused on mathematical tables used for fraction reckoning and the conversion of measures. Of particular interest has been the so-called 2÷n table, which is fundamental to Egyptian fraction reckoning. This research has been dominated from its beginnings (Neugebauer 1926) by modern mathematical analysis and remains today an area of study for mathematicians interested in the mathematics of ancient Egypt (Abdulaziz 2008; Dorce 2018). While the related issue of the development of metrological units and their usage constituted an early interest of Egyptologists (for an overview see Reineke 2014, with references to earlier studies), a substantial analysis of most systems still remains a desideratum of research, with the exception of the measures of capacity, whose analysis (Pommerening 2005) could be considered a model for research on the measurement of lengths (cf. Girndt 1996 and the unpublished dissertation Hirsch 2013) and areas. Likewise, while the few mathematical texts have been studied continuously since their first editions, with rare exceptions (e.g., Janssen 1991), no serious attempt has been made to analyze the available accounts with regard to the mathematical knowledge employed in them, and to assess the relationship between those texts and the so-called mathematical papyri.

Sources and context

Sources for ancient Egyptian calculation start with the first evidence of numbers, followed by evidence for metrological systems, and somewhat later (from the Middle Kingdom on) by texts that detail techniques and calculation aids. The latter so-called mathematical texts include tables of fraction reckoning and metrological conversions as well as mathematical problems and their solutions. Only a few mathematical texts are extant. Apart from the Rhind Mathematical Papyrus mentioned above, which is the largest extant mathematical papyrus, the Moscow Mathematical Papyrus and the Lahun and Berlin Mathematical Fragments complete the papyrological evidence for mathematical texts written in hieratic. While the Moscow Mathematical Papyrus and the Berlin and Lahun Mathematical Fragments originate from the Middle Kingdom, the Rhind Mathematical Papyrus was written in the Second Intermediate Period, but indicates at its beginning that it was copied from an earlier (Middle Kingdom?) document. In addition, there are two wooden tablets, a leather roll, and two ostraca that hold mathematical texts or parts thereof. The application of mathematical techniques can be found in ration texts, like Papyrus Berlin 10005, as well as in a papyrus from the context of construction work, Papyrus Reisner I. Last, but not least, the New Kingdom satirical letter Papyrus Anastasi I also includes sections on mathematical knowledge.

From the invention of the number system, the context in which numerical values and their calculation were situated was primarily that of controlling resources, either in administrative or in ceremonial situations. The earliest evidence of Egyptian numbers originates, along with what is considered the earliest evidence of writing, from tomb U-j in Abydos (Dreyer 1998; for a discussion of the writing found in the tomb, cf. Baines 2004 and Stauder 2010). Of the almost 200 bone and ivory tags, 45 (approximately one quarter) show numerical values. They are thought to have been attached to boxes of grave-goods, denoting quantities of items and localities (their provenience?), thus fulfilling simultaneously aspects of administration and the display of prestige.

With the further development of concepts and techniques to quantify resources, calculation techniques became an integral part of scribal education as well as scribal culture. The earliest texts that inform us about the teaching of calculation originate from the Middle Kingdom (Lahun and Berlin Mathematical Fragments and Moscow Mathematical Papyrus). The importance placed on the mathematical skills of scribes can be seen in corresponding references in the Late Egyptian Miscellanies (see Gardiner 1937; Caminos 1954; and Tacke 2001) and the section on the mathematical skills of a scribe in the satirical letter Papyrus Anastasi I. These indicate likewise the importance attached to the use of (accurate) calculations in the control of resources. It is from this knowledge and capability, and associated power, that the ancient Egyptian scribes derived their claim to distinction.

Indeed, the ancient title of the Rhind Mathematical Papyrus, the only extant (nearly) complete mathematical handbook, reads, "Method of calculating (*ḥsb*) to enter into all matters (and) to know everything, [every] darkness [...] (and) every secret," indicating the high esteem in which mathematical knowledge was held in ancient Egypt. Apart from various math problems located at the beginning of the text, the majority of math problems in the papyrus are of a concrete nature, applying to issues of administration (rations; volumes of grain; qualities and quantities of bread/beer; output of work of all kinds—e.g., agricultural produce, handwork, etc.) or construction (properties of pyramids or other structures). Throughout the papyrus the problems focus on the numerical assessment of resources.

In comparison to the scant sources from ancient Egypt, a wealth of mathematical texts from Mesopotamia have survived, which can be attributed to the choice of writing material (i.e., clay tablets; for an overview of Mesopotamian mathematics, cf. Robson 2008). A comparison of the Mesopotamian mathematical tablets with Egyptian mathematical papyri shows parallels as well as differences (Ritter 1995). Both cultures used structured collections of mathematical procedures to record and presumably teach mathematical knowledge. A first fundamental difference, however, is the number system that was employed. While Egypt's number system was a decimal system without positional notation, that of Mesopotamia was a sexagesimal system, having 60 as its base (attested from the Ur III Period onwards). As a consequence, respective areas of technical difficulties that required the use of tables varied. In Egypt, tables that facilitated fraction reckoning are well documented. In Mesopotamia, a wealth of multiplication tables have survived (Robson 2003). The more plentiful available sources for Mesopotamian mathematics also allow more detailed insights. Like the source situation in Egypt, however, the chronological distribution of Mesopotamian mathematical tablets is by no means uniform, and evidence for the first mathematical problem texts appears roughly at the same time as in Egypt.

The Development of Calculation from the Invention of Numbers to the First (Extant) Mathematical Texts

It has already been noted that the extant mathematical texts from ancient Egypt are scarce and document only selected periods of pharaonic history. While we assume that mathematical techniques were in use by the time of the Old Kingdom, no evidence for them exists. The Middle Kingdom is the best-documented period, with roughly half a dozen sources that include detailed information on calculation techniques and their underlying concepts. In later periods, evidence that calculation techniques still played an important role in scribal knowledge comes from their actual use in accounts as well as from literary texts about scribal culture: no further hieratic mathematical papyri have been discovered. The development of calculation techniques can therefore merely be provided in chronological glimpses.

Invention, form, and uses of numbers

What is considered the first evidence of writing in ancient Egypt also includes the first representations of numbers (Dreyer 1998). While the numerical tags of tomb U-j mostly show groups of horizontal or vertical lines and a few examples of a coiled rope (Dreyer 1998: pl. 28), a later specimen from Naqada also shows the combination of symbols ultimately known to represent the numerals 1, 10, and 100 on one tag (Quibell 1904 – 1905: pl. 43). These objects originate, as do many finds from ancient Egypt, from the context of tombs and seem to have been associated with grave goods, about which they perhaps conveyed numerical information. Given the predominant later setting of professional numeracy within the domain of administration, the idea that these early sources also indicate some sort of administrative effort to control grave goods seems plausible.

At the same time, the use of numbers in a representative context is documented. The macehead of King Narmer (Ashmolean Museum AN1896-1908.E.3631) constitutes the earliest evidence of all the signs larger than 100 that are used in the Egyptian representation of numbers (i.e., 1,000, 10,000, 100,000, and 1,000,000). The scene inscribed on the macehead depicts an enormous tribute to Narmer, consisting of 400,000 bulls, 1,422,000 goats, and 120,000 captives (fig. 1). Such large numbers are indicative of their representative function in displaying the power of the king. Like his macehead, Narmer's palette is illustrative: one side of the palette, depicting the king smiting his enemies, shows the number of foes to be 6,000. Thus, from as early as Dynasty 0, the symbols that were used in writing numbers are attested in a ritual and representative context—a use analogous to that of writing (cf. Stauder 2010).

A comparison with the Mesopotamian invention of a script and number system reveals similarities but also differences. The evolution of writing can be traced in more detail in Mesopotamia than in Egypt (e.g., Nissen, Damerow, and Englund 1993; Schmandt-Besserat 1996; for an overview of writing systems in Egypt, Mesopotamia, and elsewhere, cf. Houston 2004). In Mesopota-

Figure 1. Macehead of Narmer, depicting an enormous tribute of animals and captives.

mia, the attested invention and development of a number system is situated firmly in the realm of administration (Robson 2007). The use of numbers in a representative context appears scarce in Mesopotamia.

The Egyptian number system is a decimal (base 10) system without positional (place-value) notation, in which the following seven signs (𓏺𓎆𓍢𓆼𓂭𓆐𓁨) are used to write numerical values by repeating each sign as often as required (i.e., between 0 and 9 times) to represent the number, and grouping individual signs in a symmetrical arrangement. This system remained in use throughout the history of ancient Egypt. However, as of the Middle Kingdom, the sign for 1,000,000 seems to have been no longer operational (Sethe 1916: 8-10), and a multiplicative number notation to write larger numbers had been developed (for a combination of notations with a multiplicative writing for one hundred thousands followed by the old notation for lesser parts of the number, see papyrus UC 32161: Collier and Quirke 2004: 94-95). Since the Egyptian representation of numbers did not require a symbol to represent zero, areas in which zeros were used in other mathematical contexts, e.g., the expression of a balanced account, made use of other signs (for a detailed discussion cf. Hoffmann 2024).

In addition to their use in administration and political representation, there is some evidence for numbers and related concepts in the domain of Egyptian religion (Rochholz 2002). A prominent example is the mathematical operation of weighing featured in the judgment of the dead, consisting of weighing the heart of the deceased against a feather representing the goddess *Maat* to assess its quality. Similarly, some Egyptian cubit-rods found as grave goods include in their inscriptions more than simple metrological information (cf. Schlott-Schwab 1981 and Monnier, Petit, and Tardy 2016).

Fractions

The concept of Egyptian fractions became one of the most characteristic features of ancient Egypt's numeracy. The way in which fractions were represented in ancient Egypt differs fundamentally from the way we write fractions today. This difference has often led to a distorted analysis of Egyptian fraction reckoning, viewed solely through the eyes of modern mathematicians. In historical perspective, it should be noted that Egyptian fractions continued to be used in ancient Greek arithmetic and logistics (Knorr 1982 and Fowler 1992).

The evolution of fractions can be traced back at least as far as the Old Kingdom (for examples of fractional notations on the Palermo Stone and in the Abusir Papyri, cf. Ritter 1992). Two types of depictions can be differentiated: a small set of ordinary fractions (1/2, 1/3, 1/4, and 2/3) featuring special signs for each of these values, and the standard depiction for other fractions as inverses written with a dot over the representation of the respective whole number. General fractions were written additively from those elements. For example, the fraction 3/4 was written via the juxtaposition of the signs denoting 1/2 and 1/4, to be read as 1/2 + 1/4. In such representations the fractional values were arranged according to size, beginning with the largest (i.e., the inverse of the smallest integer).

The representation of fractions as sums of inverses made Egyptian fraction reckoning a technically demanding component of Egyptian mathematics, for which tables were developed. The most famous of these is the so-called 2÷*n* table, extant in the Rhind Mathematical Papyrus, as well as in one of the mathematical fragments from Lahun (UC 32159). Also noteworthy is the mathematical leather roll (BM EA10250), which lists the results of sums of fractions.

Calculation techniques: Arithmetics

Given the Rhind and Moscow Mathematical Papyri and the Lahun Mathematical Fragments, calculation techniques are comparatively well documented. Calculation techniques can be assessed on two levels: the execution of basic arithmetic operations like addition, subtraction, and multiplication (among other operations), and the more involved procedure, requiring several of these arithmetic operations to be carried out in a specific sequence indicated by listing the respective operations in the sequence (and their intermediate results), along with additional instructions.

Mathematical operations that are documented with specific terminology in the mathematical papyri are addition, subtraction, multiplication, division, doubling, halving, calculating the inverse, squaring, and extracting the square root. Of these, the actual workings of multiplications and divisions can be found in some mathematical texts either after the sequence of instructions or directly following the instruction of the particular mathematical operation, thus enabling further insights into Egyptian methods for conducting these operations. Multiplications and divisions were carried out using a scheme consisting of two columns, as shown below. For multiplications, in the initial line a dot (which was counted as one) was put in the first column, and the number to be multiplied was put in the second column (initialization). From this line the second line of the scheme was produced by doubling, halving, or decupling (multiplying by ten)—that is, any operation that could easily be computed by mental arithmetic. The third line could use either the second line (further doubling or decupling) or the first line again (or subsequently any previous line). The aim was to gain entries in the first column that would add up to the multiplier (i.e., the number with which the initial number was to be multiplied). These lines would then be marked by an initial slash (\), as seen below in the first and third lines, where 1 and 4, respectively, total the multiplier of 5. The addition of the respective entries of the second column (i.e., those lines marked with a slash) yields the result of the multiplication. For the multiplication of 2,000 times 5, this scheme would look as follows (cf. problem 52 of the Rhind Mathematical Papyrus) (fig. 2):

Figure 2. Problem 52 of the Rhind Math. Pap.

\ .	2,000	(initialization)
2	4,000	(doubling of line 1)
\ 4	8,000	(doubling of line 2)
sum	10,000	(result)

The technique of decupling is documented, as shown below, in the multiplication of 75 times 20 (cf. problem 44 of the Rhind Mathematical Papyrus) (fig. 3):

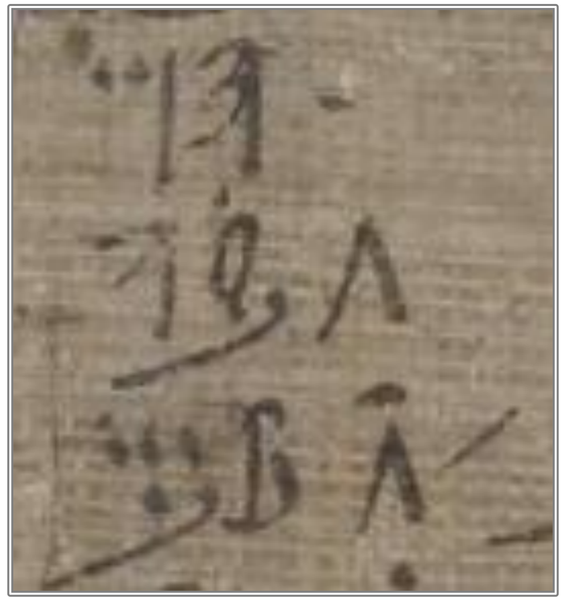

Figure 3. Problem 44 of the Rhind Math. Pap.

.	75	(initialization)
10	750	(decupling of line 1)
\ 20	1,500	(doubling of line 2) (result)

In this case, the multiplier 20 can be found directly in the last line. Therefore, the result of the multiplication (i.e., 1,500) is the entry in the second column of that line.

For divisions, the same scheme and mental arithmetic that are used for multiplications are employed. Thus, the division of 15 (the dividend) by 5 (the divisor) appears as follows (cf. problem 26 of the Rhind Mathematical Papyrus):

\ .	5	(initialization)
\ 2	10	(doubling)

The above scheme begins with an initialization by putting a dot in the first column (as in multiplications, the dot counts as 1) and the number which is operated on (the divisor) in the second column. This is doubled in the second line. While operating from column to column, the aim is to find entries in the second column that add up to the dividend. In the example above, the first two lines of the second column add up to the dividend of 15. Both lines get slashes. The result of the division is obtained by adding the slashed lines of the first column (that is, 3).

These written schemes for computing multiplications and divisions with integers (if their result also is an integer) reduce them to arithmetical operations, such as doubling or decupling, that could easily be carried out mentally in the Egyptian number system. For this reason, tables for multiplications and divisions of whole numbers, which are not documented in the hieratic documents, were superfluous. The Mesopotamian mathematical problem texts do not include the carrying out of arithmetic operations (for so-called lentil-shaped tablets containing numerical information that may be linked to Mesopotamian mathematical problem texts, cf. Robson 1999). The extant wealth of multiplication tables from Mesopotamia suggests that these were used in solving mathematical problems, whereas the Egyptian scribe included a written calculation, or an execution on another writing surface. Egyptian mathematical tables included tables for fraction reckoning as well as tables for metrological conversions.

Calculation techniques: Mathematical problems

Due to the circumstances of preservation of ancient Egyptian texts, few texts exist that originate from teaching situations. The problems found in extant mathematical texts comprise procedures for a variety of mathematical situations. Grouped from a modern mathematical perspective, they include problems we solve today by using algebraic equations (i.e., determining an unknown quantity from a specified relationship between it and its result), calculating the distribution of objects among a group of recipients (some of which one might interpret from a modern perspective as problems from number-theory), or calculating areas and volumes of various geometrical shapes. Those problems that we would call "geometric," that is, the calculation of areas or volumes, often include a sketch inscribed with numeric properties of the object ("display drawing"), along with an "in-line drawing" (representative shape of the object, which is rendered slightly larger than a hieratic sign and may also include numeric information) in the title of the problem. While these drawings are sufficiently accurate to represent the object that is the topic of the problem, they are not "technical drawings" in a modern sense, nor are they part of the solution process indicated in the problem (De Young 2009).

However, the respective grouping according to later mathematical categories (algebraic, geometric, etc.) does not render the ancient Egyptian conceptualization or mathematical structures of the problem texts. As is clear from the content and structure of the largest of these texts, the Rhind Mathematical Papyrus, the purpose of the mathematical problem texts was to teach mathematical techniques that were useful—indeed necessary—for the control of resources. Structurally, the problem texts are collections of procedures, each of which could be used to solve a specific mathematical problem, such as the calculation of the amount of work required in a specific context (e.g., of a sandal maker as found in problem 23 of the Moscow Mathematical Papyrus).

All these problems have an essentially identical structure. A problem text begins by stating the mathematical problem to be solved. After the type of problem is announced, numerical values—the data of the problem—are given, thus specifying the problem to one concrete instance or object. This is then followed by instructions (the procedure) for its solution. Each instruction usually consists of one arithmetic operation (addition, subtraction, multiplication, division, halving, squaring, extraction of the square root, calculation of the inverse of a number) and its result. The instructions always use the specific numerical values assigned to the problem. Abstract formulas, or equations with variables, did not exist. The section of instructions of a problem text prominently display the phrase *sḏm.ḫr.f* "he shall hear," both in the stating of the instructions and in the announcement of the results (for the use of this phrase in mathematical texts, cf. Imhausen 2014: 182-184; for its use in medical texts and in comparison with the phrase *sḏm.jn.f*, cf. Pommerening 2014: 13-16).

The title of a problem, its specifications, and its instructions are expressed in prose, using no mathematical symbolism. The title, as well as other parts of the text, may be accentuated by the use of red ink. Several key phrases may be used to structure the text of an individual problem (Imhausen 2014). These phrases mark the beginning or the end of individual sections of a problem text. Among these, that with the most attestations in the corpus of mathematical texts is the phrase ***tp n…*** "method of ...," used to indicate the beginning of a mathematical problem. Further key phrases (for a more comprehensive list, cf. Imhausen 2014) are ***kjj*** (indicating the beginning of a problem of the same type as the previous problem), ***mj/jr ḏd n.k*** (introducing the data of a problem), ***tp n sjtj*** (marking the beginning of a verification), and ***gmj.k nfr*** (marking the end of a problem). While the extant examples in the mathematical texts show some idiosyncratic features in phrasing or in the organization of various elements, the basic framework indicated above is (more or less) followed throughout the genre.

Despite the use of concrete numerical examples, scribes were presumably expected to be able to carry out the same procedures using different numerical values. This implies their ability to distinguish between constants of a procedure that would remain identical and data that might change (and intermediate results following from these data in the course of the procedure). Many of these problems are phrased in a daily-life setting, which may indicate that scribes used these techniques in their later work-life (for a correlation of metrological units and bread molds, cf. Pommerening 2021).

The Art of Calculation in Ancient Egyptian Culture and as Part of the Scribal Identity

From its very beginnings—the first written quantities from tomb U-j in Abydos—the sphere of numeracy in ancient Egypt was integral to the control of resources as well as to the representation of the power of the elite, who assessed and owned those resources. The content of the mathematical texts, the titles and depictions of scribes overseeing the handling of resources, and a large number of administrative texts using numerical data indicate that techniques of calculation arose and developed primarily within the context of accounting. Prior to its inclusion in the titles of two mathematical papyri, the Rhind Mathematical Papyrus and Lahun Fragment UC 32162, the term ***ḥsb*** ("to calculate") is attested in titles and depictions of accounting from as early as the Old Kingdom (fig. 4)

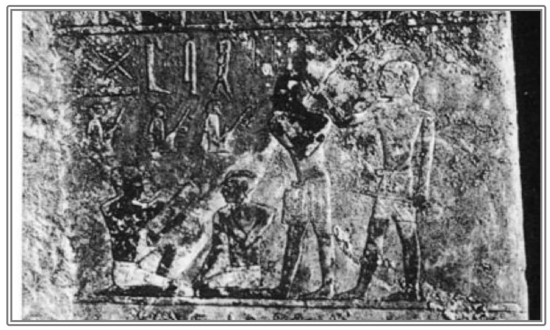

Figure 4. The Egyptian term ***ḥsb*** ("to calculate"), as shown in a relief in the mastaba of Seneb at Giza.

and indeed a cognate survives today in the Arabic word for calculation, transliterated *ḥisaab*. In accounting contexts, ***ḥsb*** designated the numerical audit of resources, and from this use derived its employment in hieratic mathematical texts, which provided training in the various numerical techniques required to carry out this form of administrative control. The mathematical texts accordingly constitute an important part of the domain of accounting in ancient Egypt. Further evidence supporting this assumption may be taken from the end of the Rhind Mathematical Papyrus, which contains three model accounts of agricultural produce. The status that was accorded accounting, this essential tool of ancient Egyptian culture, can be deduced from the elaborate title of the papyrus, which arguably would place this text in the context of a ***pr ʿnḫ*** ("house of life," the location where the composition of texts organizing and preserving knowledge of various domains may have occurred) and also from the fact that it is referenced in texts that originate from sources portraying something like a scribal identity, e.g., the satirical letter of Papyrus Anastasi I, which provides an overview of knowledge that was expected from scribes, including several examples of mathematical problems (as well as the implicit knowledge of mathematics in the description of scribal tasks during an expedition).

The use of calculation in administration also explains the special status it held as a means of implicit justification. In the course of Egyptian history, the association of mathematical techniques with the claim that their results were inherently just led to mathematical practices also being applied to document justice in other domains and contexts, such as in the judgment of the dead and in the Teaching of Amenemope.

Bibliographic Notes

Access to reliable literature on ancient Egyptian mathematics is not straightforward. It must be kept in mind that earlier literature from the domain of the history of mathematics was often written under the assumption that mathematics is universal in regard to time and place. This resulted in the expression of (our perception of) ancient mathematical knowledge using modern mathematical language, employing, for example, algebraic formulas that were not part of an ancient tradition and do not assist us in gaining insight into ancient calculation techniques. Recent research has shown the need for us to acknowledge the forms in which ancient mathematical knowledge was recorded and also the concepts underlying that knowledge, which may well be different from their modern successors (compare, for example, Ritter 1995 and 2010). Despite this awareness, even recent semi-popular literature on Egyptian mathematics often continues to use inappropriate renderings of the content of Egyptian mathematical texts, such as algebraic equations, for example. An introduction into the methodological issues of various ancient scientific disciplines can be found in Imhausen and Pommerening (2016). A concise introduction into Egyptian mathematics can be found in Ritter (2004). For a detailed overview of Egyptian mathematics from its beginnings until the Ptolemaic and Roman Periods, see Imhausen (2016). The latter also provides references to the vast literature that exists on specific mathematical problems. The relationship between mathematics and architecture is detailed in Rossi (2004). While most of the ancient Egyptian mathematical texts await re-edition, the Lahun Fragments have been edited relatively recently (Collier and Quirke 2004: 71-96); despite their fragmentary character, they provide a good overview of mathematical problems and tables included in the hieratic material. More recent translations of some mathematical texts published at the beginning of the twentieth century can be found in Imhausen (2007) (in English) and Imhausen (2020) (in German).

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Figure 1. Macehead of Narmer, depicting an enormous tribute of animals and captives. Oxford, Ashmolean Museum AN1896-1908.E.3631. (Photograph by the author.)

Figure 2. Problem 52 of the Rhind Mathematical Papyrus. (From Robbins and Shute 1987: Plate 16.)

Figure 3. Problem 44 of the Rhind Mathematical Papyrus. (From Robbins and Shute 1987: Plate 15.)

Figure 4. The Egyptian term *ḥsb* (“to calculate”), as shown in a relief in the mastaba of Seneb at Giza. (Junker 1941: Plate VI.)