Rethinking the Prisoner's Dilemma: How Quantum Games Lead to Classical Gains

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The Prisoner's Dilemma is a popular thought experiment in game theory where two prisoners are each given the option to confess to their crimes or remain silent. Variable jail times are associated with the four possible outcomes (Figure 1). Given that the two players cannot interact or collude with one another, each player is incentivised to maximize their own gain, meaning they act without altruistic tendencies.[1] Surprisingly, classical economic theory predicts that both players will always choose to confess, landing both with five years of jail time. However, upon inspecting Figure 1, one notices that it is more favorable for neither player to confess, leading to both inmates receiving two years of jail time. The decision to confess in exchange for a prolonged jail time contradicts all natural reasoning, but it is exactly what classical game theory predicts any one of us would do given the choice. The inability to maximise the outcomes of the players in the prisoner's dilemma is a major limitation of classical game theory, which can be resolved through the addition of "quantum strategies."

A BRIEF INTRODUCTION TO THE CLASSICAL MODEL

Game theory is a field of economics and mathematics concerned with the modelling and study of social interactions to explain social dilemmas. Given a predetermined collection of strategies and their corresponding outcomes, game theory allows for the prediction of the decisions of any participants in such interactions or dilemmas.[2]

In the classical model, both players have a dominant strategy for each game, corresponding to a player maximizing their personal gain independent of the other player's decision. A player's dominant strategy can be determined by viewing the table in Figure 1; specifically, the dominant strategy for both A and B separately is to confess, with a corresponding average jail time of three years versus an average of five years for not confessing.[2]

However, people often change their behaviors based on what they expect others' actions to be. When a player is aware of their opponent's optimal moves, they will vary theirs accordingly; this leads to a Nash equilibrium. Equilibrium is a self-perpetuating situation where, even after seeing the other player's move, the person still opts to remain in their current square since they have chosen their optimal move.[2,3] Specifically, in a Nash equilibrium, neither player gains by a singular horizontal or vertical movement from the square containing the equilibrium (see Figure 1). Looking back at Figure 1, if Player A knew that Player B was going to confess, Player A would also confess (five versus eight years of jail time). Similarly, if Player A knew that Player B would not confess, their best option would be to confess (one versus two years of jail time). The Nash equilibrium reached in the prisoners' dilemma occurs when both confess, as this minimizes the potential losses of each player given the power of the other player.[2]

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This Nash equilibrium is not the socially optimal outcome; in fact, we know that both players not confessing would be beneficial to all parties involved. If asked to choose whichever option they wanted, neither player would choose the Nash equilibrium of five years of jail time. However, unsure of the other player's move, the Nash equilibrium is the best, least risky response, even if it does not necessarily have the highest yield. We can even see that both players confessing leads to the longest combined jail time of any option.

player A/player B	Don't confess (Q)	Confess (D)
Don't confess (Q)	2 years/2 years (QQ)	8 years/1 year (QD)
Confess (D)	1 year/8 years (DQ)	5 years/5 years (DD)

Figure 1: The outcomes of the Prisoner's Dilemma, with each unshaded cell containing the jail time associated with Player A and Player B's decisions.

This type of game is both a simultaneous game, in which the players are not aware of the other's move, and a non-zero-sum game, in which the players are both not aware of the other's move and can achieve a better outcome if they were able to collaborate.[3,5]

LIMITATIONS OF CLASSICAL GAME THEORY

In the Prisoner's Dilemma, one person's action greatly affects the other player. However, if the two players cannot coordinate their actions, they both end up with a lower payoff than they would separately desire. This is a large limitation of classical theory, as it assumes the requirement of direct communication to improve their results from the suboptimal Nash equilibrium. Additionally, the removal of societal preferences in the classical model results in models that are unrepresentative of human behaviour. These classical boundaries can be resolved through the application of quantum strategies, specifically quantum entanglement.[4,5]

INTRODUCING QUANTUM THEORY

In the classical game, we can write "confessing" and "not confessing" as two different states, Q and D, respectively. Each square in Figure 1 also represents a state that can be written as QQ, QD, DQ, and DD, where the order of the two letters corresponds to the decisions made by Player A and Player B, respectively. Q and D are determinate states at any given moment in time, with a determinate state being one with a well-defined outcome upon measurement. For example, if you roll a fair dice, the outcome is determinate; it will be one of the six sides.[6]

However, quantum states are indeterminate, meaning they can be represented by a superposition—a linear combination of classically determinant states. For example, aQ + bD is a linear combination, a sum of states where a and b are complex numbers corresponding to the probability of finding the particle in state Q or D.[6]

When an ideal measurement is made on this superposition state, an observer will see the system either in state Q or D. The system will collapse into one of these states upon measurement. Now, rather than definitively knowing the state of the system, the observer considers the probability of each state.[6]

A property unique to quantum mechanics that arises in these systems is entanglement. Not all quantum states will be entangled, but to aid in solving problems associated with classical theory, we will only consider entangled systems.[4] Entangled states are non-local states; this means that no matter the spatial distance between these states, they will affect one another. If we separated two entangled particles to opposite ends of the earth and made a measurement on particle 1, we could instantaneously calculate the measurement of particle 2. Classically, this would be impossible as information can only travel at the speed of light, and hence we could not immediately know the effect one particle has on the other.[4]

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This provides a new way to approach game theory; with both participants having access to entangled states, they could now have additional information about the other's decision, even without communication. Thus, the players can now more effectively coordinate their strategies, resulting in a better outcome for all parties involved.

QUANTUM PRISONER'S DILEMMA

By applying quantum game theory to the Prisoner's Dilemma (Figure 1), we can improve the players' outcomes. By assuming there is no entanglement, Player A's probabilities do not depend on B's, leading A to confess and vice versa. This reflects the known outcomes for the classical scenario. However, by adding entanglement to the system, the Nash equilibrium may change to a more favorable outcome.[5]

The amount of entanglement of the system directly correlates with the new Nash equilibrium reached. Figure 2 reflects an experiment comparing the strength of entanglement to the payoff of Player A (called Alice) in the prisoner's dilemma.

Figure 2 can be split into three regions, which are separated by the two critical points gamma-1 = 0.63 and gamma-2 = 0.79. Each region corresponds to a different Nash equilibrium as a direct result of the different strengths of entanglement.

Consider an analysis on the points L1, L2, and L3, which are arbitrarily chosen, with one in each different region. Upon analysis of the data, for sufficiently small entanglement (L1), the quantum outcomes match those classically predicted. For the intermediate range (L2), the new Nash equilibriums are DQ and QD. Above the highest threshold (L3), the new Nash equilibrium is not confessing for both; this is a better outcome for both players than the classical Nash equilibrium of both

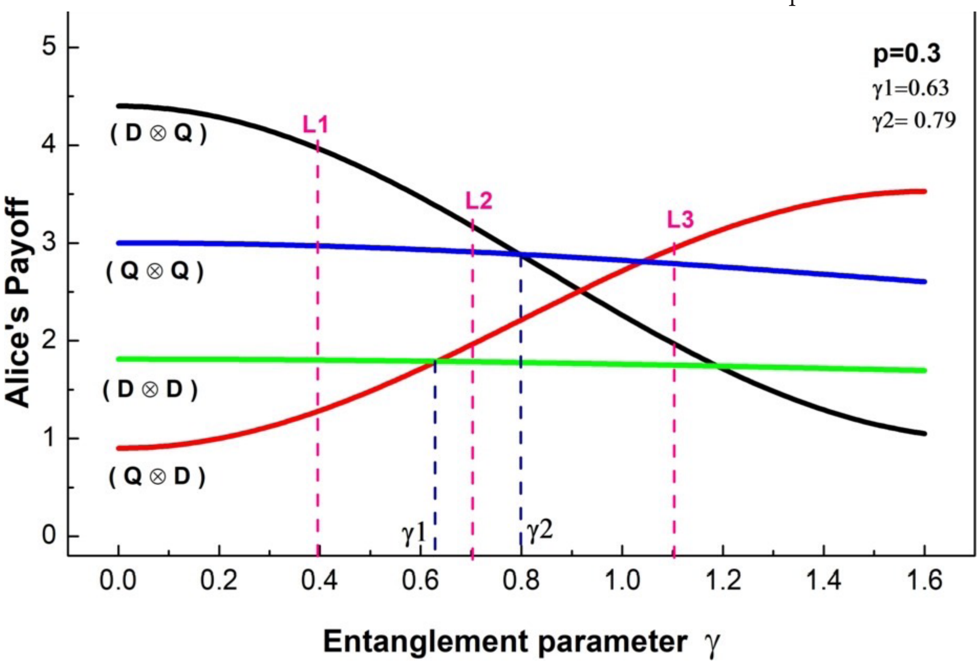

Figure 2[7]: A representation of how Player A's (called Alice in the graph above) optimal choice in the prisoner's dilemma is dependent on the strength of the quantum entanglement. In this graph, the magnitude of the entanglement parameter (gamma) is plotted against A's payoff, with the DQ, QQ, DD, and QD states being the black, blue, green, and red lines, respectively.

player A/player B	Don't confess (Q)	Confess (D)
Don't confess (Q)	10/10 (QQ)	0/5 (QD)
Confess (D)	5/0 (DQ)	1/1 (DD)

Figure 3: The outcomes of Prisoners' Dilemma, each unshaded cell containing the payoff (benefit) associated with Player A and Player B's decisions. Similar to Figure 1, but a payoff matrix states the gain to each player, where a higher payoff is better, corresponding to a lower jail time.[7]

confessing, showing that access to quantum states in the prisoner's dilemma allows for improved outcomes.[7]

Given the graphical data at each L point, to find the new Nash equilibrium, one can form a payoff matrix. Figure 3 corresponds to the payoff matrix for L1. L2 and L3 will have similar forms with different payoff values.[8]

APPLICATIONS OF QUANTUM GAME THEORY

The uses and applications of quantum game theory are often related to considering time-evolving probability distributions rather than a definitive set of outcomes. This is particularly useful in reflecting the constantly evolving world that we live in, compared to the great number of approximate restrictions placed on systems by classical economists.

For example, classical game theory assumes that players care only about their own payoff and that individuals do not have personal opinions on reciprocity and altruism. The entanglement of systems that arises with quantum games better reflects the effects of players on one another, corresponding to a more realistic scenario where we would expect two players who have committed a crime together to know one another. Impressively, these quantum strategies do not require the use of quantum computers and can be implemented solely through classical devices, allowing for the utilization of these improved strategies in daily life.[7]

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IMAGE REFERENCE

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